Nucleon structure from 2+1-flavor dynamical DWF lattice QCD at nearly physical pion mass

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RBRC lunch talk, November 14, 2013

RBC and UKQCD collaborations have been generating dynamical Domain-Wall Fermions (DWF) ensembles:

- good chiral and flavor symmetries,

that allowed us do a lot of good pion and kaon physics as well as nucleon.

We are now much closer to physical pion mass with large volume, than the previous sets of ensembles:

- light, $m_\pi \sim 171$ and 248 MeV, quarks ($m_{ud}a = 0.001$ and 0.0042, and $m_{\rm res}a \sim 0.002$),
- a large, $(4.6\text{fm})^3$, volume ($a^{-1} \sim 1.371(10)$ GeV),

made possible by Iwasaki + dislocation suppressing determinant ratio (DSDR) gauge action.

Here we report the current status of our nucleon calculations, by

- Meifeng Lin, Yasumichi Aoki, Tom Blum, Taku Izubuchi, Chulwoo Jung, SO, Shoichi Sasaki, Eigo Shintani, Takeshi Yamazaki, ...

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RBC/UKQCD $N_f = 2+1$ dynamical DWF ensembles, with good flavor and chiral symmetries:

- extrapolations to chiral and continuum limits are disentangled,
- with fully non-perturbative renormalizations.
- Also, reweighing allows calculations with exact strange mass.

With Iwasaki gauge action at $a^{-1} = 1.75(4)$ and $2.31(4)$ GeV [1] with volumes larger than 2.7 fm across,

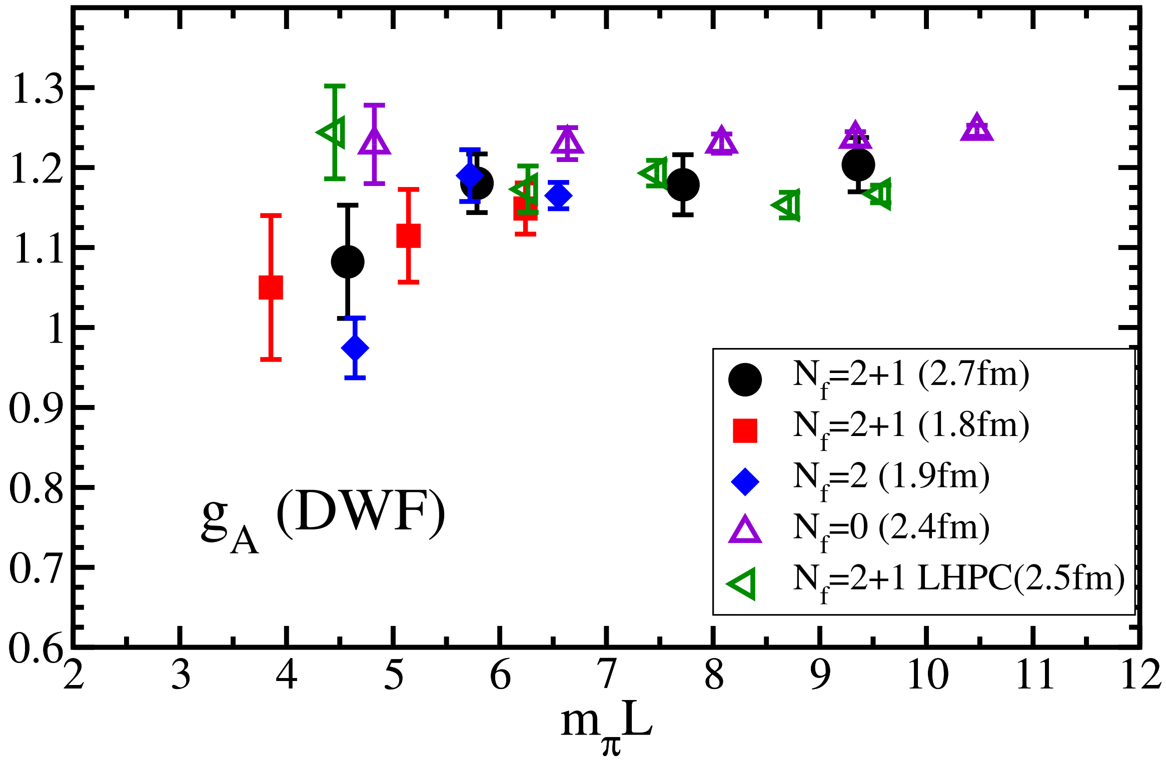
- $m_\pi = 0.67$, 0.56, 0.42 and 0.33 GeV, $m_N = 1.55$, 1.39, 1.22 and 1.15 GeV.
- $f_\pi = 124(2)(5)$ MeV, $f_K/f_\pi = 1.204(7)(25)$; $m_{\rm s}^{\overline{\rm MS}(2{\rm GeV})} = 97(3)$ MeV, $m_{\rm ud}^{\overline{\rm MS}(2{\rm GeV})} = 3.6(2)$ MeV,
- Constraints on CKM matrix: $B_K^{\overline{\rm MS}(3{\rm GeV})} = 0.529(5)_{\rm stat}(15)_\chi(2)_{\rm FV}(11)_{\rm NPR}$, K_{l3} $f_+(0) = 0.964(5)$, ...
- Chiral perturbation is useless from this heavy mass range, $m_\pi \sim 300$ MeV: e.g. NLO $\sim 0.5\times$LO.

Systematics arising from too heavy pion mass was dominating. We need lighter pion:

- new Iwasaki+DSDR action[2], $a^{-1} \sim 1.371(10)$ GeV, $m_\pi \sim 250$ and 170 MeV, $L \sim 4.6$ fm.

[1] URL: http://link.aps.org/doi/10.1103/PhysRevD.83.074508, DOI: 10.1103/PhysRevD.83.074508;
[2] URL: http://link.aps.org/doi/10.1103/PhysRevD.87.094514, DOI: 10.1103/PhysRevD.87.094514.

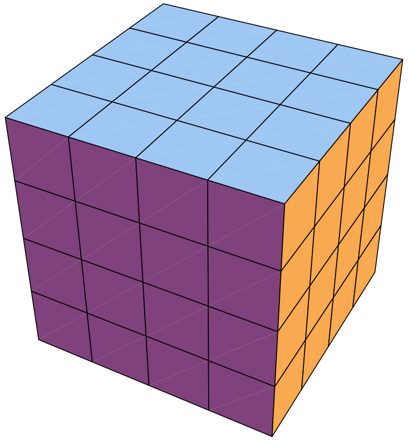
Nucleon structure: In Lattice 2007 Takeshi Yamazaki reported unexpectedly large finite-size effect in axial charge, $g_A/g_V = 1.2701(25)$:

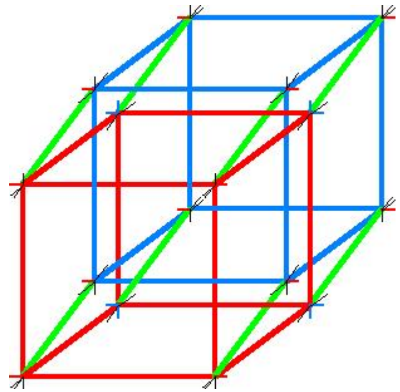


- Heavier quarks: almost consistent with experiment, no discernible quark-mass dependence.
- Lighter quarks: finite-size sets in as early as $m_\pi L \sim 5$, appear to scale in $m_\pi L$:
- If confirmed, first concrete evidence of pion cloud surrounding nucleons.

Or is this caused by excited-state contamination?
Calculations at lighter pion mass in larger volume should help.

Lattice: 4D simple hyper-cubic lattice, $L_0L_1L_2L_3$, Euclidean





site: $s = (n_0n_1n_2n_3)$, $0 \le n_i \le L_i - 1$ $(i = 0, 1, 2, 3)$.

link: $l = (s, \mu)$, $\mu \in \{0, 1, 2, 3\}$, connects s and $s + \hat{\mu}$.

constant separation (lattice constant) a between neighboring sites.

Taking $a \to 0$ through asymptotic scaling gives exact continuum physics.

Dynamical variables:

quark: $q(s)$, defined on site and forms basis of fundamental (3) representation of SU(3),

gluon: $U(s, \mu) = \exp(ig \int_s^{s+\hat{\mu}} A_\mu(y) \mathrm{d}y_\mu) \in \mathrm{SU}(3)$, now a group element defined on link.

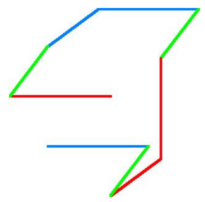
There are many other ways to define lattice (eg. random lattice) with different advantages, but the way q, U and G are defined is basically the same.

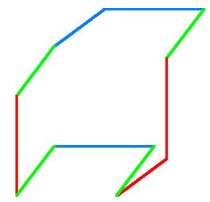
Gauge transformation: $G(s) \in \mathrm{SU}(3)$, defined on site, maps quarks and gluons

$$q(s) \mapsto G(s)q(s) \;\text{ and }\; U(s,\mu) \mapsto G(s)U(s,\mu)G(s+\hat{\mu})^{-1}.$$

Gauge invariant objects (QCD action, observables):

- Quark: $\bar{\psi}(x)U(x,\mu)U(x+\hat{\mu},\nu)...U(y-\hat{\rho},\rho)\psi(y), \mapsto \bar{\psi}(x)\underline{G^{-1}(x)G(x)}U(x,\mu)\underline{G^{-1}(x+\hat{\mu})G(x+\hat{\mu})}U(x+\hat{\mu},\nu)...U(y-\hat{\rho},\rho)\underline{G^{-1}(y)G(y)}\psi(y)$.





- Gluon, $\mathrm{Tr}[U(x,\mu)U(x+\hat{\mu},\nu)...U(x-\hat{\rho},\rho)] \mapsto \mathrm{Tr}[\underline{G(x)}U(x,\mu)\underline{G^{-1}(x+\hat{\mu})G(x+\hat{\mu})}U(x+\hat{\mu},\nu)...U(x-\hat{\rho},\rho)\underline{G^{-1}(x)}]$.

Action: $S_{\mathrm{QCD}}[U,q,\bar{q}] = S_{\mathrm{gluon}}[U] + S_{\mathrm{quark}}[U,q,\bar{q}]$, must respect gauge invariance:

gluon part: such as $S_{\mathrm{gluon}}[U] = \frac{6}{g^2}\sum_s \sum_{\mu<\nu} \Box(s,\mu,\nu)$, gives $-\frac{1}{2}\mathrm{Tr}\, G^{\mu\nu}G_{\mu\nu}$ as $a \to 0$ and $g \to 0$,

- with plaquette $\Box(s,\mu,\nu) = 1 - \frac{1}{3}\mathrm{Re}\,\mathrm{Tr}U(s,\mu)\,U(s+\hat{\mu},\nu)U(s+\hat{\nu},\mu)^{-1}U(s,\nu)^{-1}$.

quark part: $S_{\mathrm{quark}}[U,q,\bar{q}] = \sum_{s,s'} \bar{q}(s)M[U](s,s')q(s')$, which should give $\bar{q}(i\gamma^\mu D_\mu - m)q$,

- with $M[U](s,s')$ describing quark propagation between sites s and s'.

Expectation values of any gauge-invariant observable: $\langle O\rangle = N^{-1}\int[\mathrm{d}U][\mathrm{d}q][\mathrm{d}\bar{q}]O[U,q,\bar{q}]\exp(-S_{\mathrm{QCD}}[U,q,\bar{q}])$,

or by integrating over the quark Grassmann variables: $N'^{-1}\int[\mathrm{d}U](\det M[U])\exp(-S_{\mathrm{gluon}}[U])$.
It is often convenient to use effective action: $\tilde{S}[U] = S_{\mathrm{gluon}}[U] - \mathrm{Tr}\log M[U]$.

Finite lattice and compact SU(3) assures finite $\langle O\rangle$, evaluated with importance sampling of $\exp(-S)$.

Continuum limit is well defined through asymptotic freedom: consider an observable O with mass dimension,

- the expectation value is described as $\langle O\rangle = a^{-1}f(g)$ with some dimensionless function $f(g)$ of dimensionless coupling g.
- Renormalizability of the theory means the cutoff dependence should vanish, $\dfrac{d\langle O\rangle}{da} \to 0$, as $a \to 0$, or

$$f(g) - f'(g)\left(a\frac{dg}{da}\right) = \beta(g)f'(g) + f(g) \to 0.$$

- This $(df/f = -dg/\beta)$ is easily solved to give: $\langle O\rangle a \propto \exp\left(-\int^g \dfrac{dh}{\beta(h)}\right)$, or

$$\langle O\rangle a \propto (g^2 b_0)^{-\frac{b_1}{2b_0^2}} e^{-\frac{1}{2b_0 g^2}}[1+O(g^2)],$$

where $\beta(g) \equiv -a\dfrac{dg}{da} = -b_0 g^3 - b_1 g^5 + O(g^7)$ is perturbatively well known.

Chiral symmetry:

- Invariance under global $U(N_f)$ transformations, $q \mapsto \exp(i\theta)q$, $\exp(i\theta'\gamma_5)q$, $\exp(i\alpha^a \frac{\lambda^a}{2})q$ and $\exp(i\beta^a \frac{\lambda^a}{2}\gamma_5)q$.

- Should be preserved in the absence of $m\bar{q}q$, like $U(N_f)_L \times U(N_f)_R = SU(N_f)_V \times SU(N_f)_A \times U(1)_V \times U(1)_A$.

- In fact spontaneously broken for light normal quarks, $m_u \sim m_d \sim 0$, $\langle \bar{u}u + \bar{d}d \rangle \neq 0$.
- Important for Nambu-Goldstone pion, PCAC, etc, $m_\pi^2 f_\pi^2 = m_q \langle \bar{q}q \rangle$.

However, difficult to maintain on regular lattices.

Naive lattice fermion action, with $M_{xy} = \frac{1}{2}a^{D-1}\sum_\mu \gamma_\mu[\delta_{x+\hat{\mu},y} - \delta_{x-\hat{\mu},y}]$, leads to a propagator $\Delta(p) = a[\gamma_\mu \sin(p_\mu a)]^{-1}$, which has 2^D poles at $p_\mu = 0$ or π/a: for $D = 4$, there are $2^4 = 16$ flavors/tastes instead of one.

Shifting of one component of p_μ, such as $\tilde{p}_\mu = p_\mu - \pi/a$, acts like

$$\gamma_\mu \sin(p_\mu a) = -\gamma_\mu \sin(\tilde{p}_\mu a)$$

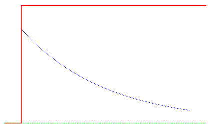
so the chirality $\pm$ states are paired.

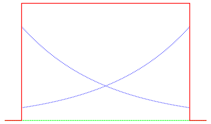
Nielsen and Ninomiya theorem: doubling inevitable (chirality $\pm$ states are paired) for a regular lattice and local, hermitian, and translationally invariant action.

Domain-wall fermions[3]: introduce a 5-th dimension, s, and define a 5D Dirac operator: $D = \gamma_\mu \partial_\mu + \gamma_5 \partial_s + m(s)$,

- With a monotonic $m(s)$ with $m(s=0)=0$, a 4D chiral modes emerge: $\psi_\pm(x,s) = u_p(x)\phi_\pm(s)\chi_\pm$.
- 4D Dirac plane wave u_p and γ_5 eigenstate, $\gamma_5\chi_\pm = \pm\chi_\pm$, indicate the s-dependence,

$$[\pm\partial_s + m(s)]\phi(s) = 0, \quad \text{or} \quad \phi(s) \propto \exp[\mp \int_0^s ds' m(s')],$$

pinned at the $s=0$ wall, and exponentially decay to $\pm s$ direction. 

- On a finite lattice, two walls, with a pair of $\pm$ chiralities mix. 
- No problem for a vector theory like QCD[4]: mixing exponentially suppressed, described by $m_{\rm res}$.

RIKEN-BNL-Columbia (RBC) Collaboration proved DWF works very well for QCD:

- light hadron mass spectrum,
- electroweak transitions among light hadrons (such as f_π, f_K, B_K and ϵ'/ϵ),

unlike conventional Wilson and staggered fermions.

[3] D.B. Kaplan, Phys. Lett. B288, 342 (1992), hep=lat/9206013.
[4] Y. Shamir, Nucl. Phys. B406, 90 (1993), hep-lat/9303005; V. Furman and Y. Shamir, Nucl. Phys. B439, 54 (1995), hep-lat/9405004; and references cited therein.

QCDSP and QCDOC computers: dedicated for lattice QCD calculations.

QCDSP: completed in 1998, 600 (RBRC) and 400 (Columbia) GFlops configurations

- based on commercial DSP
- assisted by custom designed 4D hypercubic nearest-neighbor communication
- 10$ per MFlops

Demonstrated the use of DWF in (quenched) lattice QCD

- Chiral and flavor symmetries and associated ease in non-perturbative renormalizations,
- hadron spectroscopy: masses and decay constants,
- hadron matrix elements: B_K, ϵ'/ϵ, K_{l3}, nucleon form factors and structure functions.

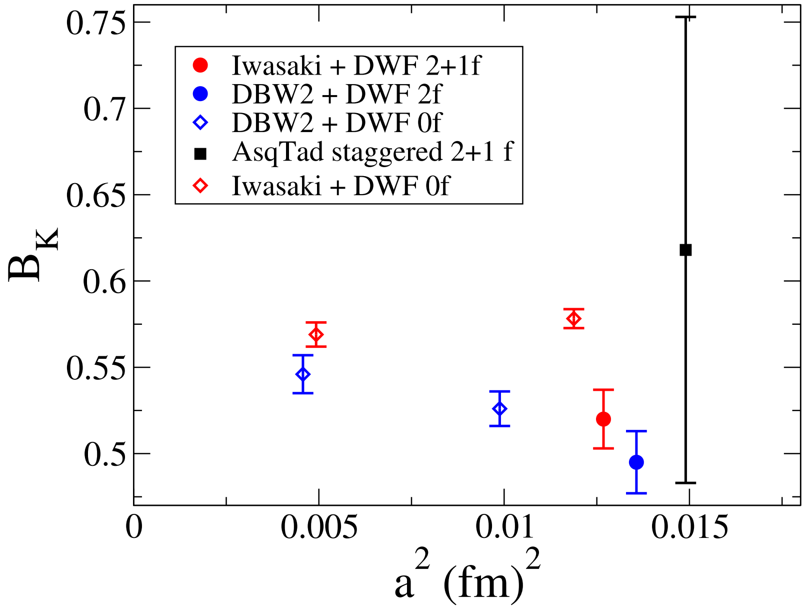
QCDOC: complete in 2005, 10 TFLops configurations in RBRC, BNL and Edinburgh.

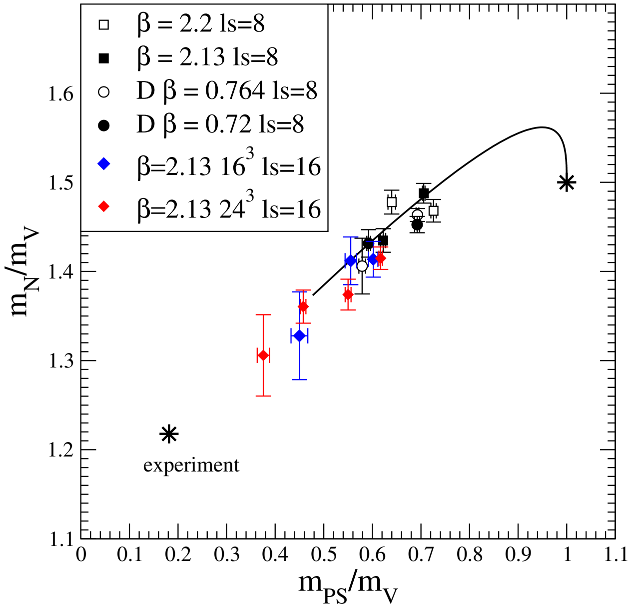
- based on system on a chip technology,
- a QCDSP card was shrunk to be a QCDOC chip, with custom-designed 6D hypercubic communicaitons,
- 1$ per MFlops.

Used for realistic (2+1)-flavor dynamical DWF lattice QCD.

Evolved into BG/L, P and Q $\sim$ QCDCQ.

RBC/UKQCD $N_f = 2+1$ dynamical DWF ensembles: $a^{-1} = 1.75(4)$ and $2.31(4)$ GeV with volumes larger than 2.7 fm across, using QCDOC,





Chiral and continuum limit with good flavor and chiral symmetries:

- $f_\pi = 124(2)(5)$ MeV, $f_K/f_\pi = 1.204(7)(25)$; $m_s^{\overline{\rm MS}(2{\rm GeV})} = 97(3)$ MeV, $m_{\rm ud}^{\overline{\rm MS}(2{\rm GeV})} = 3.6(2)$ MeV,
- Constraints on CKM matrix: $\hat{B}_K^{\overline{\rm RGI}} = 0.75(3)$, K_{l3} $f_+(0) = 0.964(5)$, ...

Chiral systematics now dominates the error. We need lighter pion:

- newer ensembles at $a^{-1} \sim 1.371(10)$ GeV, $m_\pi \sim 250$ and 170 MeV, $L \sim 4.6$ fm[5].

Contribute to determining SM parameters from pion/kaon calculations.

[5]URL: http://link.aps.org/doi/10.1103/PhysRevD.87.094514, DOI: 10.1103/PhysRevD.87.094514.

Nucleon form factors, measured in elastic scatterings or β decay or muon capture:

$$\langle p|V_\mu^+(x)|n\rangle = \bar{u}_p\left[\gamma_\mu F_V(q^2) + \frac{i\sigma_{\mu\lambda}q_\lambda}{2m_N}F_T(q^2)\right]u_n e^{iq\cdot x},$$

$$\langle p|A_\mu^+(x)|n\rangle = \bar{u}_p\left[\gamma_5\gamma_\mu F_A(q^2) + \gamma_5 q_\mu F_P(q^2)\right]u_n e^{iq\cdot x}.$$

$$F_V = F_1, F_T = F_2; G_E = F_1 - \frac{q^2}{4m_N^2}F_2, G_M = F_1 + F_2.$$

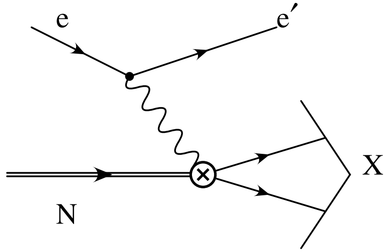
Related to mean-squared charge radii, anomalous magnetic moment, $g_V = F_V(0) = G_{\rm Fermi}\cos\theta_{\rm Cabibbo}$, $g_A = F_A(0) = 1.2701(25)g_V$, Goldberger-Treiman relation, $m_N g_A \propto f_\pi g_{\pi NN}$, ... determine much of nuclear physics.

On the lattice, with appropriate nucleon operator, for example, $N = \epsilon_{abc}(u_a^T C\gamma_5 d_b)u_c$, ratio of two- and three-point correlators such as $\dfrac{C_{\rm 3pt}^{\Gamma,O}(t_{\rm sink},t)}{C_{\rm 2pt}(t_{\rm sink})}$ with

$$C_{\rm 2pt}(t_{\rm sink}) = \sum_{\alpha,\beta}\left(\frac{1+\gamma_t}{2}\right)_{\alpha\beta}\langle N_\beta(t_{\rm sink})\bar{N}_\alpha(0)\rangle,$$

$$C_{\rm 3pt}^{\Gamma,O}(t_{\rm sink},t) = \sum_{\alpha,\beta}\Gamma_{\alpha\beta}\langle N_\beta(t_{\rm sink})O(t)\bar{N}_\alpha(0)\rangle,$$

give a plateau in t for a lattice bare value $\langle O\rangle$ for the relevant observable, with appropriate spin ($\Gamma = (1+\gamma_t)/2$ or $(1+\gamma_t)i\gamma_5\gamma_k/2$) or momentum-transfer (if any) projections.

Deep inelastic scatterings 

$: \left|\frac{\mathcal{A}}{4\pi}\right|^2 = \frac{\alpha^2}{Q^4} l^{\mu\nu} W_{\mu\nu},\ W^{\mu\nu} = W^{[\mu\nu]} + W^{\{\mu\nu\}}$

- unpolarized: $W^{\{\mu\nu\}}(x,Q^2) = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) F_1(x,Q^2) + \left(P^\mu - \frac{\nu}{q^2} q^\mu\right)\left(P^\nu - \frac{\nu}{q^2} q^\nu\right) \frac{F_2(x,Q^2)}{\nu}$,
- polarized: $W^{[\mu\nu]}(x,Q^2) = i\epsilon^{\mu\nu\rho\sigma} q_\rho \left(\frac{S_\sigma}{\nu}(g_1(x,Q^2) + g_2(x,Q^2)) - \frac{q\cdot S P_\sigma}{\nu^2} g_2(x,Q^2)\right)$,

with $\nu = q\cdot P$, $S^2 = -M^2$, $x = Q^2/2\nu$.

Moments of the structure functions are accessible on the lattice:

$$
\begin{aligned}
2\int_0^1 dx x^{n-1} F_1(x,Q^2) &= \sum_{q=u,d} c_{1,n}^{(q)}(\mu^2/Q^2, g(\mu))\, \langle x^n\rangle_q(\mu) + \mathcal{O}(1/Q^2),\\
\int_0^1 dx x^{n-2} F_2(x,Q^2) &= \sum_{f=u,d} c_{2,n}^{(q)}(\mu^2/Q^2, g(\mu))\, \langle x^n\rangle_q(\mu) + \mathcal{O}(1/Q^2),\\
2\int_0^1 dx x^{n} g_1(x,Q^2) &= \sum_{q=u,d} e_{1,n}^{(q)}(\mu^2/Q^2, g(\mu))\, \langle x^n\rangle_{\Delta q}(\mu) + \mathcal{O}(1/Q^2),\\
2\int_0^1 dx x^{n} g_2(x,Q^2) &= \frac{1}{2}\frac{n}{n+1} \sum_{q=u,d} [e_{2,n}^{q}(\mu^2/Q^2, g(\mu))\, d_n^q(\mu) - 2e_{1,n}^{q}(\mu^2/Q^2, g(\mu))\, \langle x^n\rangle_{\Delta q}(\mu)] + \mathcal{O}(1/Q^2)
\end{aligned}
$$

- c_1, c_2, e_1, and e_2 are the Wilson coefficients (perturbative),
- $\langle x^n\rangle_q(\mu)$, $\langle x^n\rangle_{\Delta q}(\mu)$ and $d_n(\mu)$ are forward nucleon matrix elements of certain local operators,
- so is $\langle 1\rangle_{\delta q}(\mu) = \langle P,S|\bar{\psi} i\gamma_5 \sigma_{\mu\nu}\psi|P,S\rangle$ which may be measured by polarized Drell-Yan and RHIC Spin.

Unpolarized (F_1/F_2): on the lattice we can measure: $\langle x\rangle_q$, $\langle x^2\rangle_q$ and $\langle x^3\rangle_q$.

$$\frac{1}{2}\sum_s \langle P,S|\mathcal{O}^q_{\{\mu_1\mu_2\cdots\mu_n\}}|P,S\rangle = 2\langle x^{n-1}\rangle_q(\mu)[P_{\mu_1}P_{\mu_2}\cdots P_{\mu_n} + \cdots - (\text{trace})]$$

$$\mathcal{O}^q_{\mu_1\mu_2\cdots\mu_n} = \bar{q}\left[\left(\frac{i}{2}\right)^{n-1}\gamma_{\mu_1}\overleftrightarrow{D}_{\mu_2}\cdots\overleftrightarrow{D}_{\mu_n} - (\text{trace})\right]q$$

Polarized (g_1/g_2): on the lattice we can measure: $\langle 1\rangle_{\Delta q}$ (g_A), $\langle x\rangle_{\Delta q}$, $\langle x^2\rangle_{\Delta q}$, d_1, d_2, $\langle 1\rangle_{\delta q}$ and $\langle x\rangle_{\delta q}$.

$$-\langle P,S|\mathcal{O}^{5q}_{\{\sigma\mu_1\mu_2\cdots\mu_n\}}|P,S\rangle = \frac{2}{n+1}\langle x^n\rangle_{\Delta q}(\mu)[S_\sigma P_{\mu_1}P_{\mu_2}\cdots P_{\mu_n} + \cdots - (\text{traces})]$$

$$\mathcal{O}^{5q}_{\sigma\mu_1\mu_2\cdots\mu_n} = \bar{q}\left[\left(\frac{i}{2}\right)^{n}\gamma_5\gamma_\sigma\overleftrightarrow{D}_{\mu_1}\cdots\overleftrightarrow{D}_{\mu_n} - (\text{traces})\right]q$$

$$\langle P,S|\mathcal{O}^{[5]q}_{[\sigma\{\mu_1]\mu_2\cdots\mu_n\}}|P,S\rangle = \frac{1}{n+1}d^q_n(\mu)[(S_\sigma P_{\mu_1} - S_{\mu_1}P_\sigma)P_{\mu_2}\cdots P_{\mu_n} + \cdots - (\text{traces})]$$

$$\mathcal{O}^{[5]q}_{[\sigma\mu_1]\mu_2\cdots\mu_n} = \bar{q}\left[\left(\frac{i}{2}\right)^{n}\gamma_5\gamma_{[\sigma}\overleftrightarrow{D}_{\mu_1]}\cdots\overleftrightarrow{D}_{\mu_n} - (\text{traces})\right]q$$

and transversity (h_1):

$$\langle P,S|\mathcal{O}^{\sigma q}_{\rho\nu\{\mu_1\mu_2\cdots\mu_n\}}|P,S\rangle = \frac{2}{m_N}\langle x^n\rangle_{\delta q}[(S_\rho P_\nu - S_\nu P_\rho)P_{\mu_1}P_{\mu_2}\cdots P_{\mu_n} + \cdots - (\text{traces})]$$

$$\mathcal{O}^{\sigma q}_{\rho\nu\mu_1\mu_2\cdots\mu_n} = \bar{q}[\left(\frac{i}{2}\right)^{n}\gamma_5\sigma_{\rho\nu}\overleftrightarrow{D}_{\mu_1}\cdots\overleftrightarrow{D}_{\mu_n} - (\text{traces})]q$$

Higher moment operators mix with lower dimensional ones: Only $\langle x\rangle_q$, $\langle 1\rangle_{\Delta q}$, $\langle x\rangle_{\Delta q}$, d_1, and $\langle 1\rangle_{\delta q}$ can be measured with $\vec{P} = 0$.

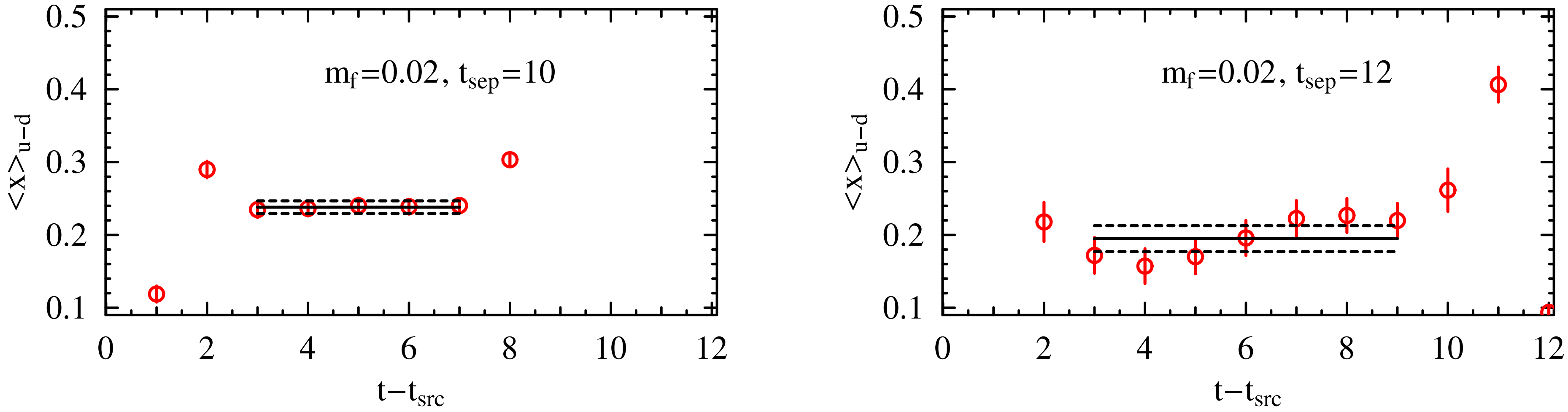
Previous RBC and RBC+UKQCD calculations addressed two important sources of systematics:

- Time separation between nucleon source and sink,
- Spatial volume.

And though not explicitly addressed yet, a better understanding of quark mass dependence is necessary.

Source/sink time separation:

- If too short, too much contamination from excited states, but if too long, the signal is lost.



- In an earlier RBC 2-flavor DWF study at $a^{-1} \sim 1.7$ GeV, separation of 10 or 1.1 fm appeared too short.

No source or sink is purely ground state:

$$e^{-E_0 t}|0\rangle + A_1 e^{-E_1 t}|1\rangle + ...,$$

resulting in dependence on source-sink separation, $t_{\rm sep} = t_{\rm sink} - t_{\rm source}$,

$$\langle 0|O|0\rangle + A_1 e^{-(E_1-E_0)t_{\rm sep}}\langle 1|O|0\rangle + ...$$

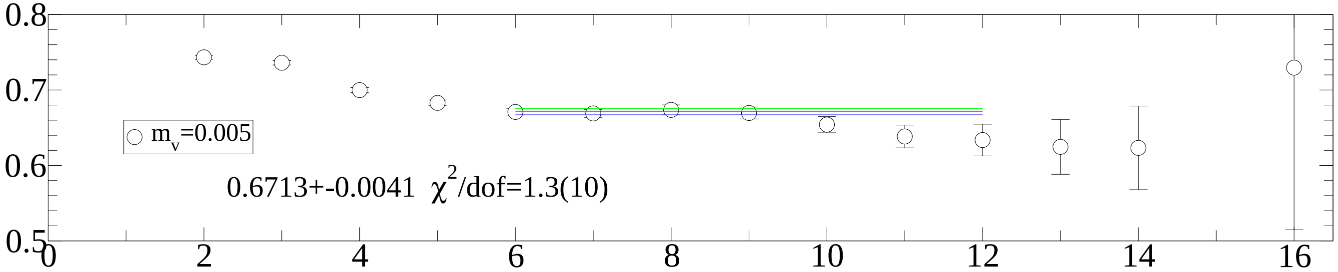
Any conserved charge, $O = Q$, $[H, Q] = 0$, is insensitive because $\langle 1|Q|0\rangle = 0$.

- g_V is clean,
- g_A does not suffer so much, indeed we never detected this systematics,
- structure function moments are not protected, so we saw the problem.

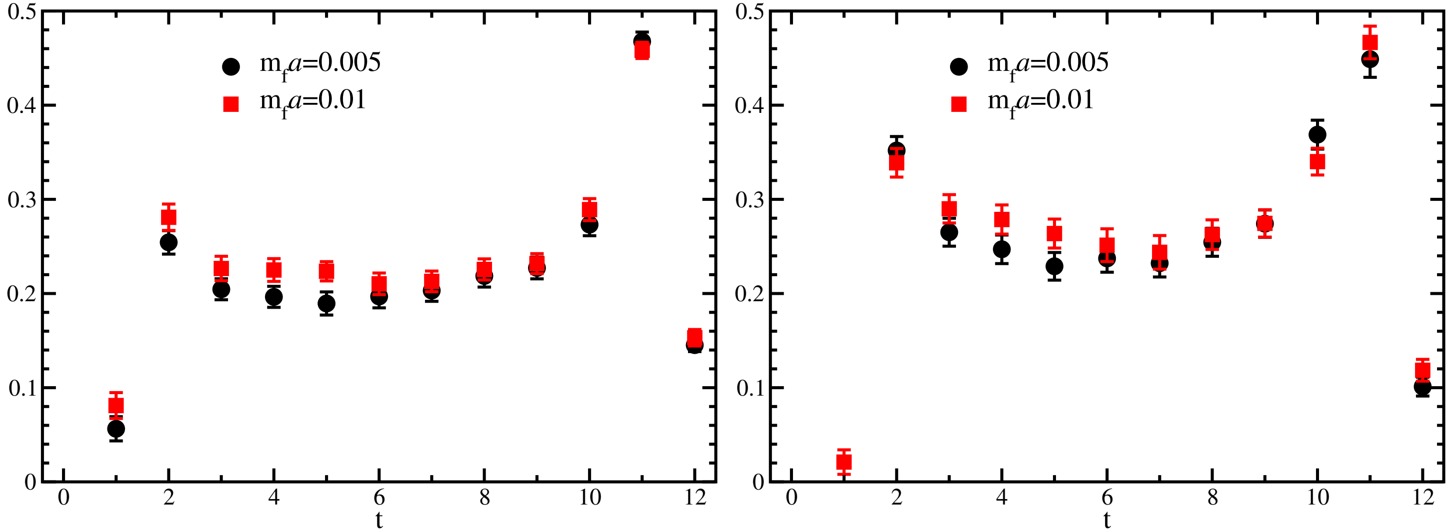
We can optimize the source so that A_1 is small, and we take sufficiently large $t_{\rm sep}$.

In the previous (2+1)-flavor study we choose separation 12 or 13 , ∼1.4 fm:

Mass signal ($m_f = 0.005$):

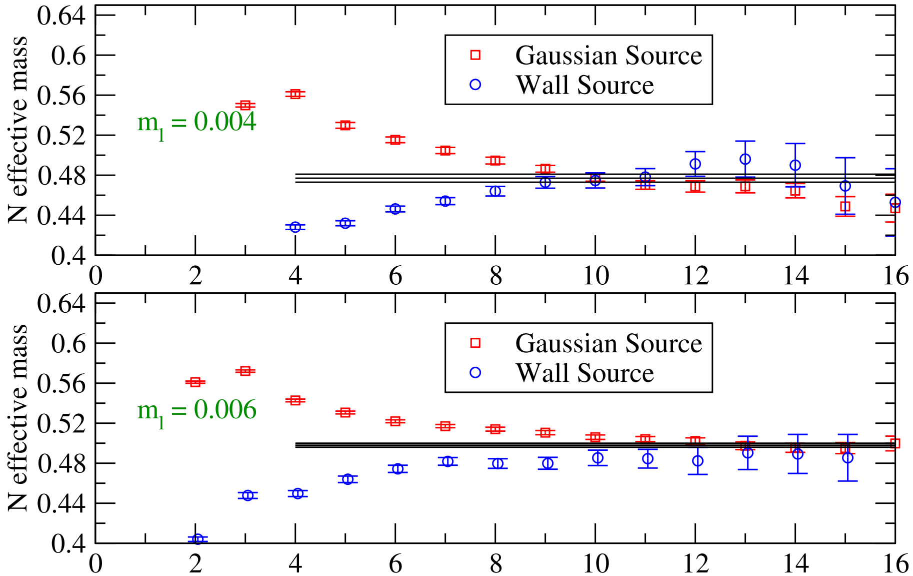


Bare three-point functions: $\langle x \rangle_{u-d}$ (left) and $\langle x \rangle_{\Delta u - \Delta d}$ (right), for $m_f = 0.005$ (black •) and 0.01 (red □):



In the present study we like to do at least as good, hopefully better: separation of 9 lattice units or longer.

On the other hand, with RBC+UKQCD 2.2-GeV (2+1)-flavor dynamical DWF ensemble:



2-state fits suggest excited-state survives $t_{\text{sink}} \geq 9$.

LHP analysis of vector form factors with $t_{\text{sep}} = 12$ or 1 fm agree with RBC+UKQCD 1.7-GeV results.

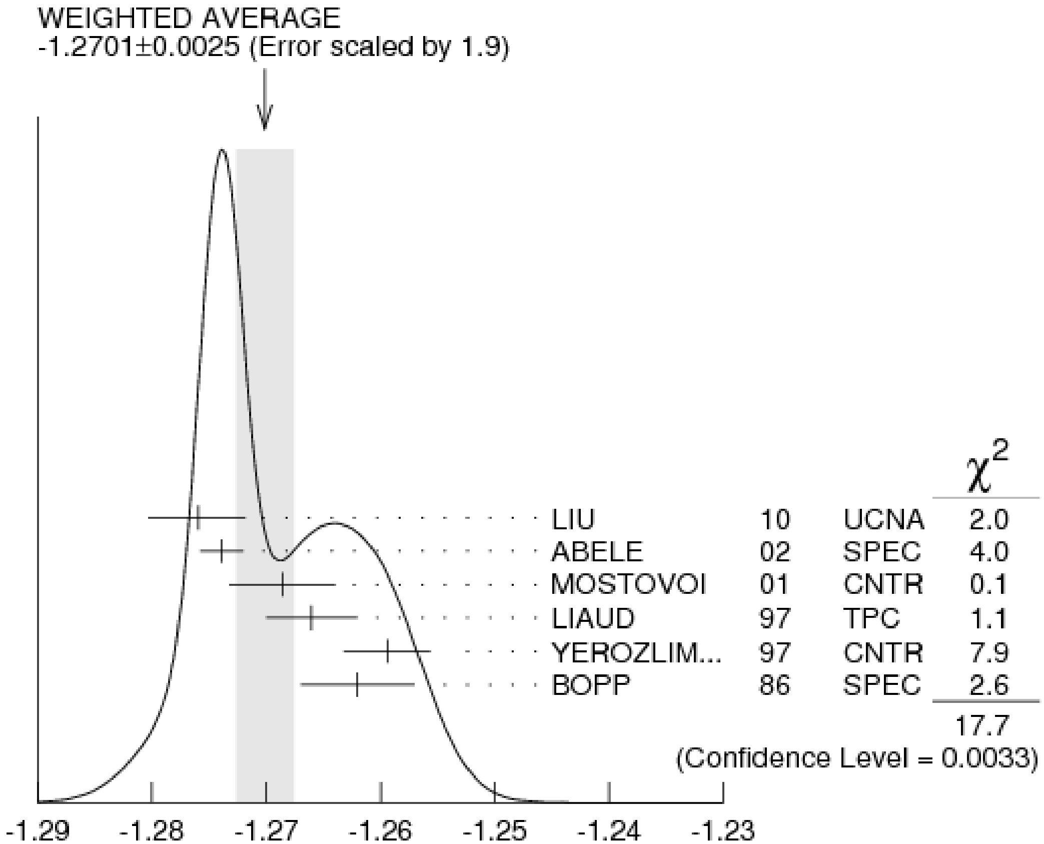
Vector current is less sensitive: conserved charge cannot tell excited-state contamination, for example.

Can we go shorter, ∼1 fm, separation, in spite of our lighter masses?

- Perhaps with better tuned source and sink smearing?
- Would be good as we have to fight growing error, $\sim \exp(-3m_\pi t)$.

LHP now seem to agree with us that their choice was too short.

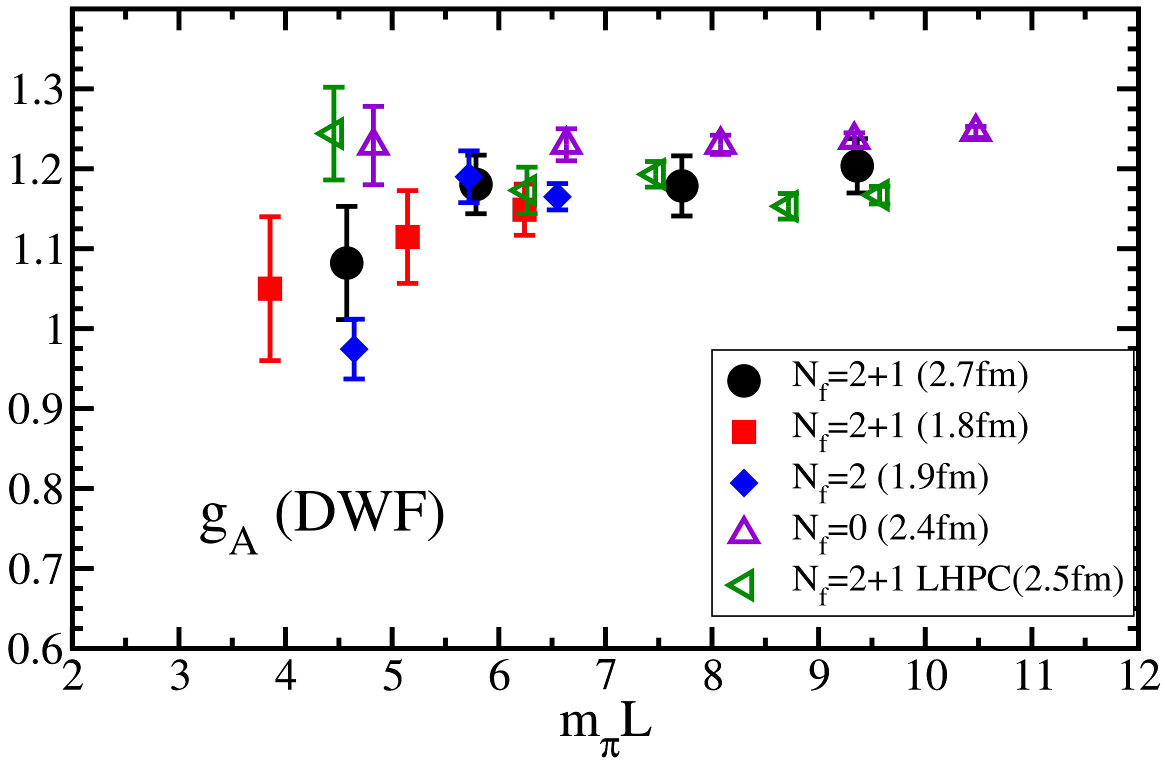
Spatial volume: let's look at nucleon isovector axial charge, g_A/g_V=1.2701(25),



Experimental value has been almost monotonically increasing since Maurice Goldhaber's first measurement.

Lattice calculations appeared to follow the same path, but, Spatial volume. In Lattice 2007 Takeshi Yamazaki reported unexpectedly large finite-size effect:

- in axial charge, $g_A/g_V = 1.2701(25)$, measured in neutron β decay, decides neutron life.



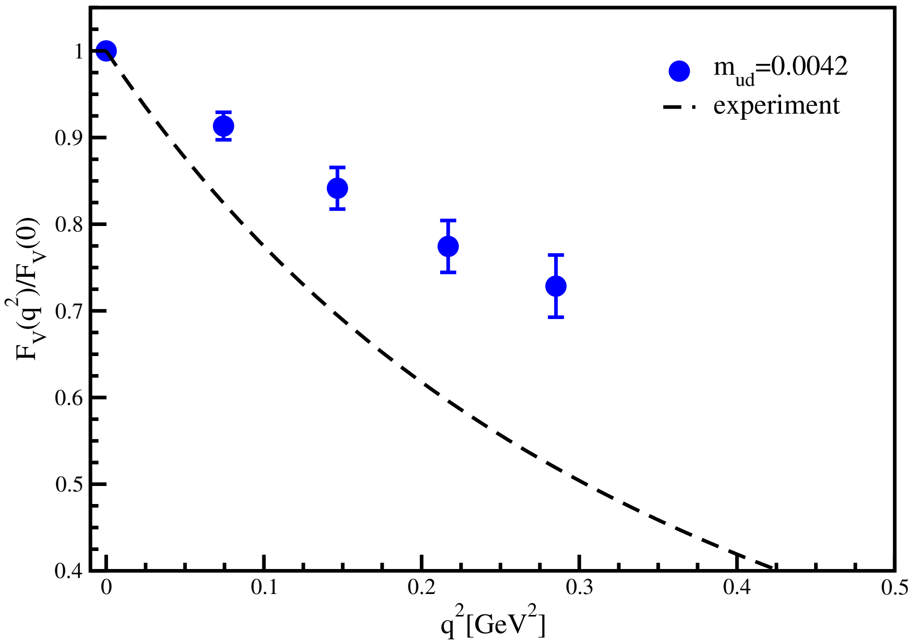
- Heavier quarks: almost consistent with experiment, no discernible quark-mass dependence.
- Lighter quarks: finite-size sets in as early as $m_\pi L \sim 5$, appear to scale in $m_\pi L$:
- If confirmed, first concrete evidence of pion cloud surrounding nucleons.

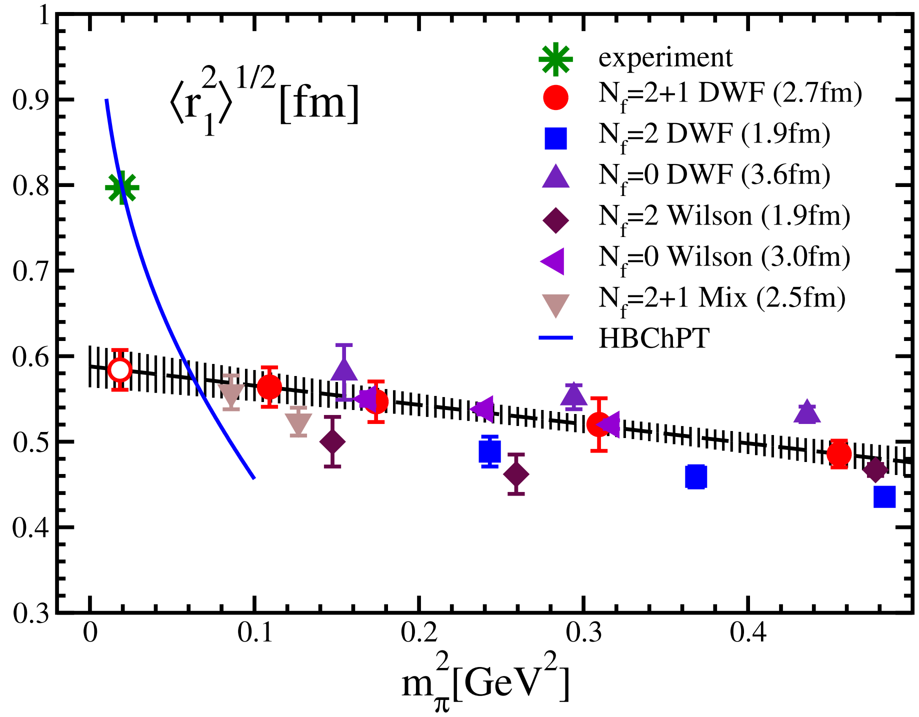
Many in the past pointed out this is a fragile quantity as pion mass is set light: Adkins+Nappi+Witten, Jaffe, Kojo+McLerran+Pisarski, ...

RBC/UKQCD(2+1)-flavor, Iwasaki+DWF dynamical, $a^{-1} = 1.75(4)$ GeV, $m_{\rm res} = 0.00315(2)$, $m_{\rm strange} = 0.04$,

- $m_\pi = 0.67$, 0.56, 0.42 and 0.33 GeV; $m_N = 1.55$, 1.39, 1.22 and 1.15 GeV,

Dirac form factor of the isovector vector current,





much too small rms radius,
no sign for logarithmic divergence anticipated from HBχPT.

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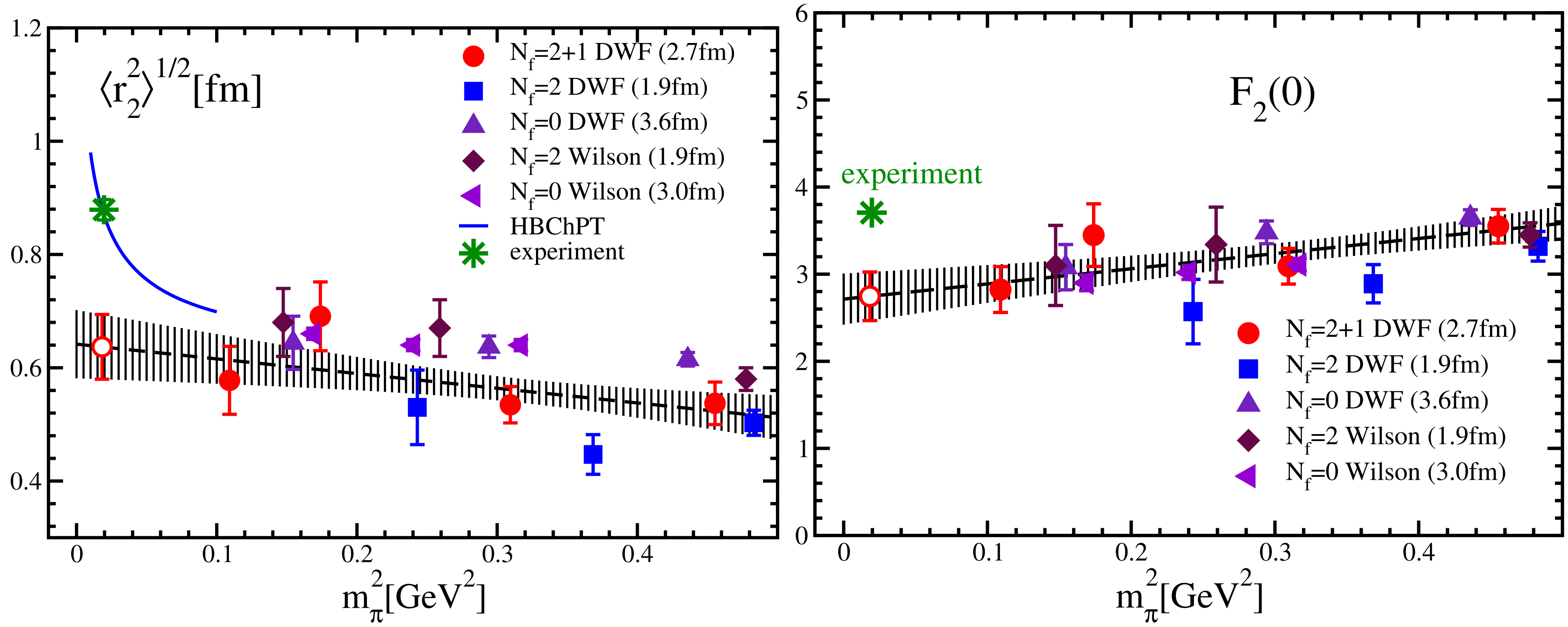
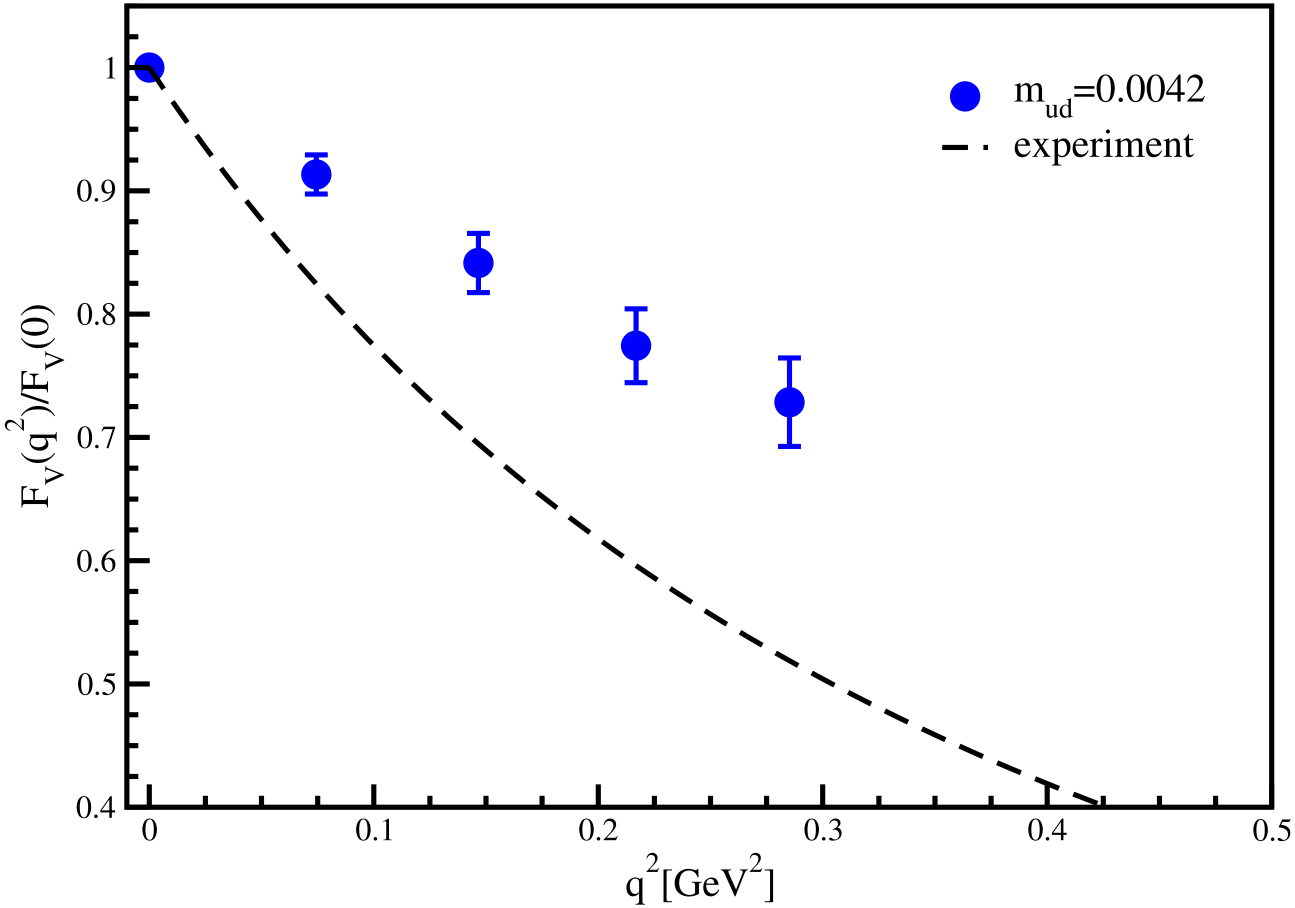


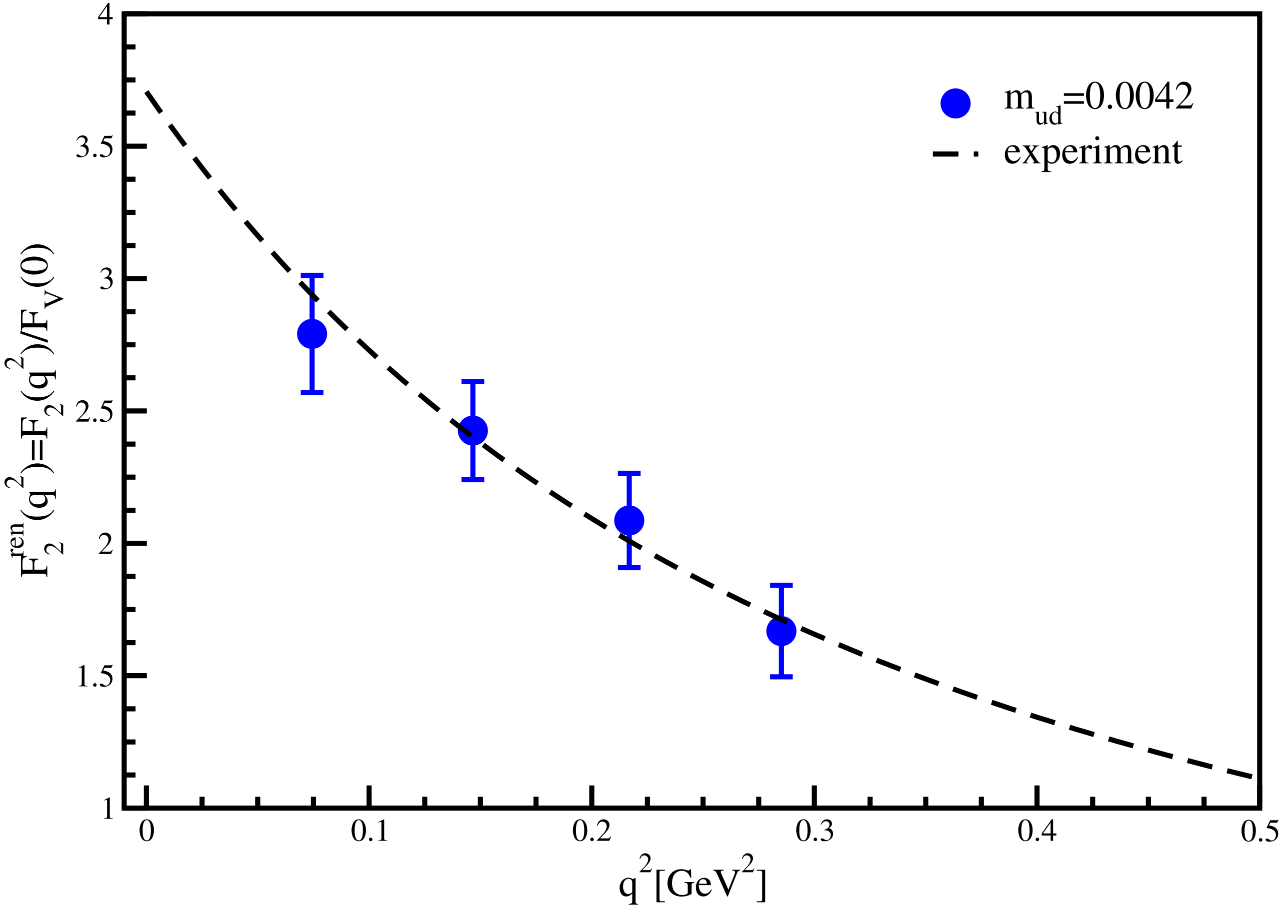
much too small rms radius,
no sign for logarithmic divergence anticipated from HBχPT,
perhaps better agreement with experiment for magnetic moment.

Dirac form factor:



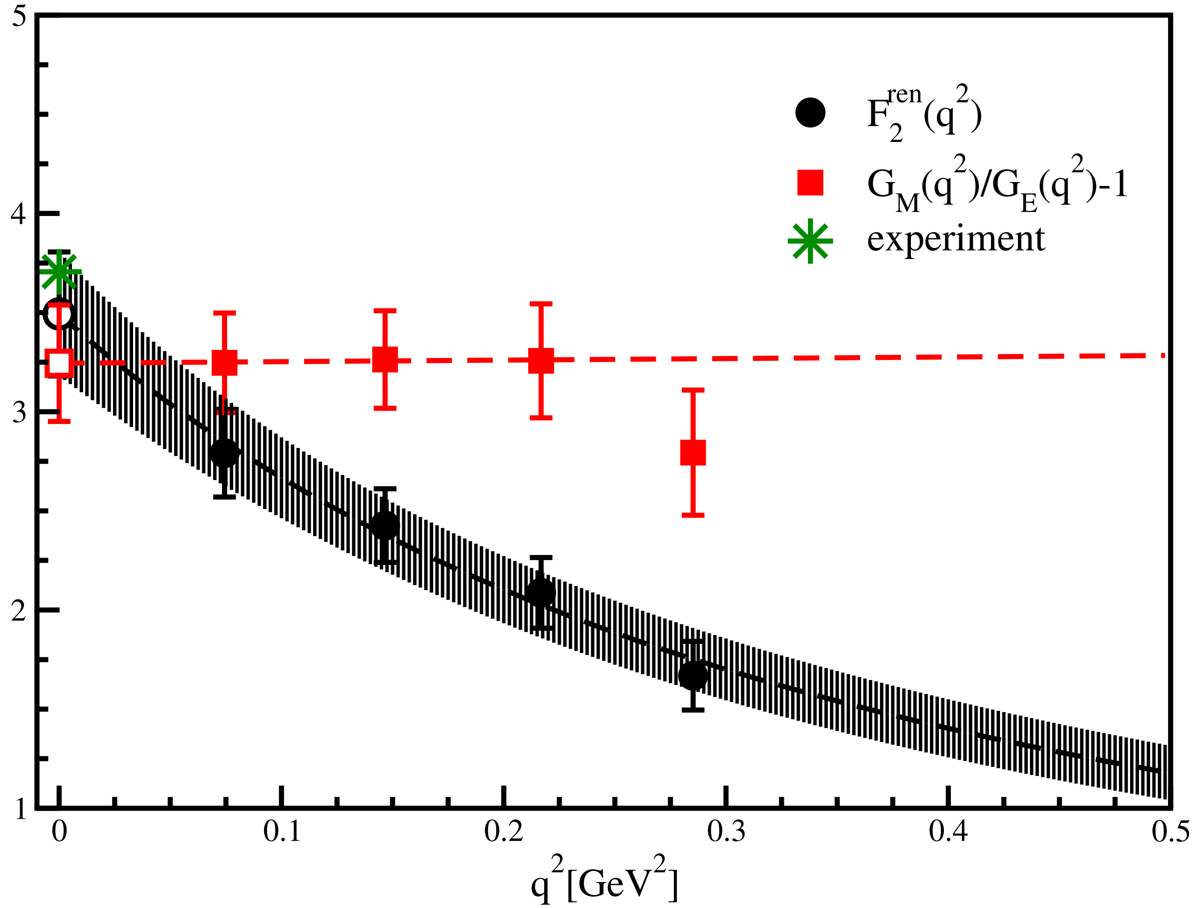
Dirac rms radius $\langle r_1^2 \rangle$ is clearly underestimated.

Pauli form factor:



Pauli rms radius $\langle r_2^2 \rangle$ now seems almost consistent with experiment, albeit with large statistical errors.

Anomalous magnetic moment: seems in agreement with experiment.

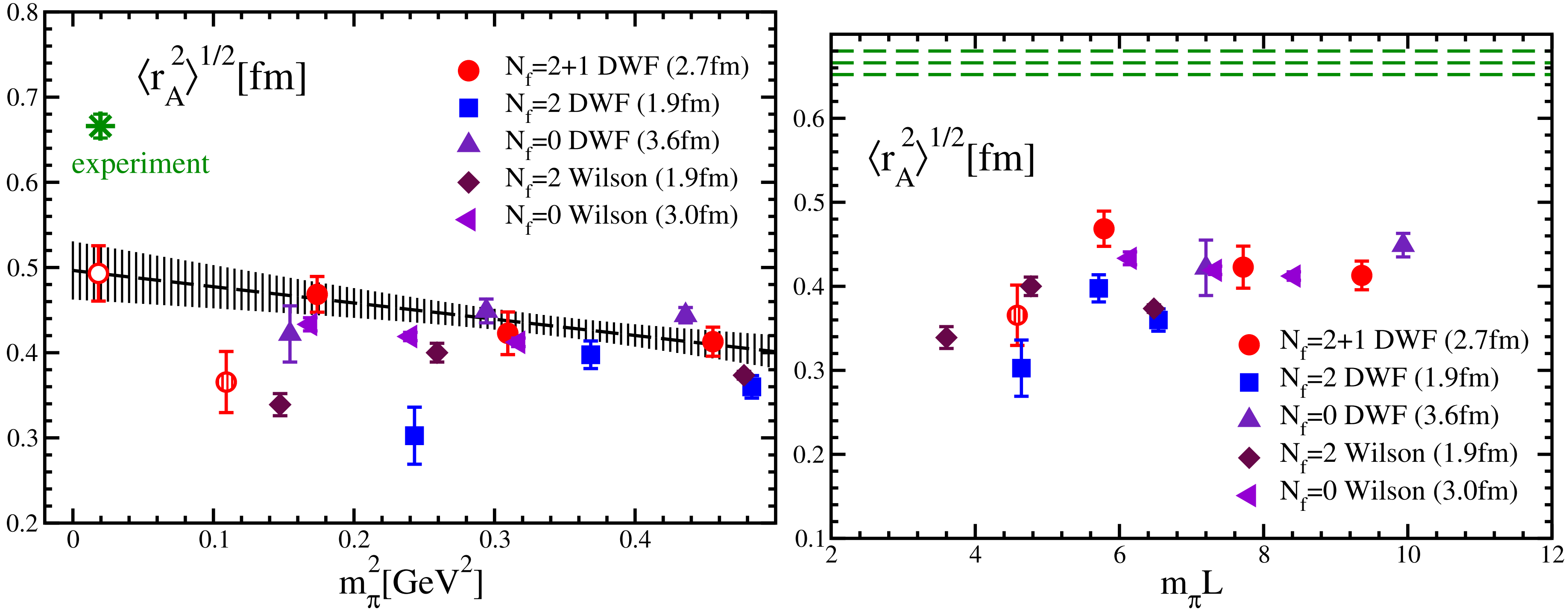


The q^2 extrapolation toward $q^2 = 0$ for either $F_2^{\mathrm{ren}}(q^2)$ or $G_M(q^2)/G_E(q^2) - 1$ for $m_{\mathrm{ud}} = 0.0042$. For $F_2^{\mathrm{ren}}(q^2)$, the dipole form is applied for all four data points, while a simple linear extrapolation with three lowest q^2 data points is used for $G_M(q^2)/G_E(q^2) - 1$ thanks to its mild q^2 dependence.

RBC/UKQCD(2+1)-flavor, Iwasaki+DWF dynamical, $a^{-1} = 1.75(4)$ GeV, $m_{\rm res} = 0.00315(2)$, $m_{\rm strange} = 0.04$,

- $m_\pi = 0.67$, 0.56, 0.42 and 0.33 GeV; $m_N = 1.55$, 1.39, 1.22 and 1.15 GeV,

Isovector axialvector form factor from the axial-vector current,

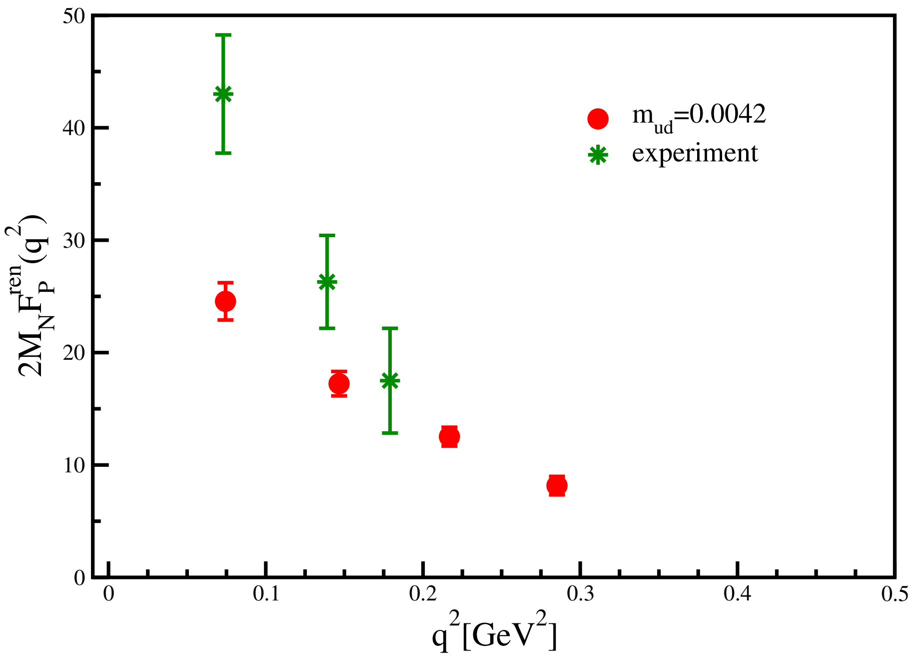


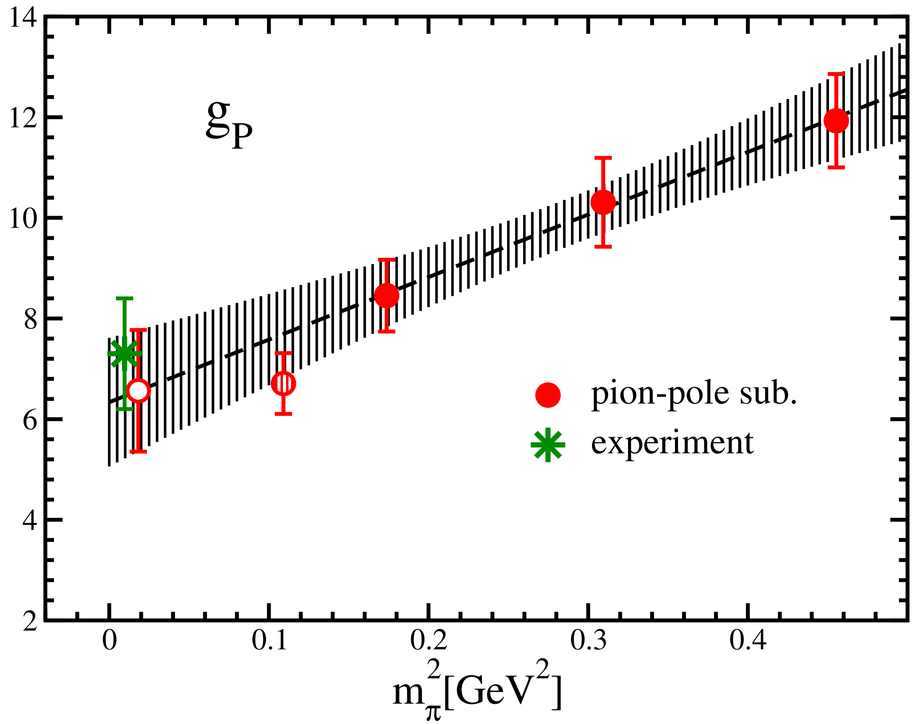
much too small rms radius,
similar dependence on $m_\pi L$ as g_A/g_V.

RBC/UKQCD(2+1)-flavor, Iwasaki+DWF dynamical, $a^{-1} = 1.75(4)$ GeV, $m_{\rm res} = 0.00315(2)$, $m_{\rm strange} = 0.04$,

- $m_\pi = 0.67$, 0.56, 0.42 and 0.33 GeV; $m_N = 1.55$, 1.39, 1.22 and 1.15 GeV,

Isovector pseudo scalar form factor from the axial-vector current,





perhaps better agreement with experiments.

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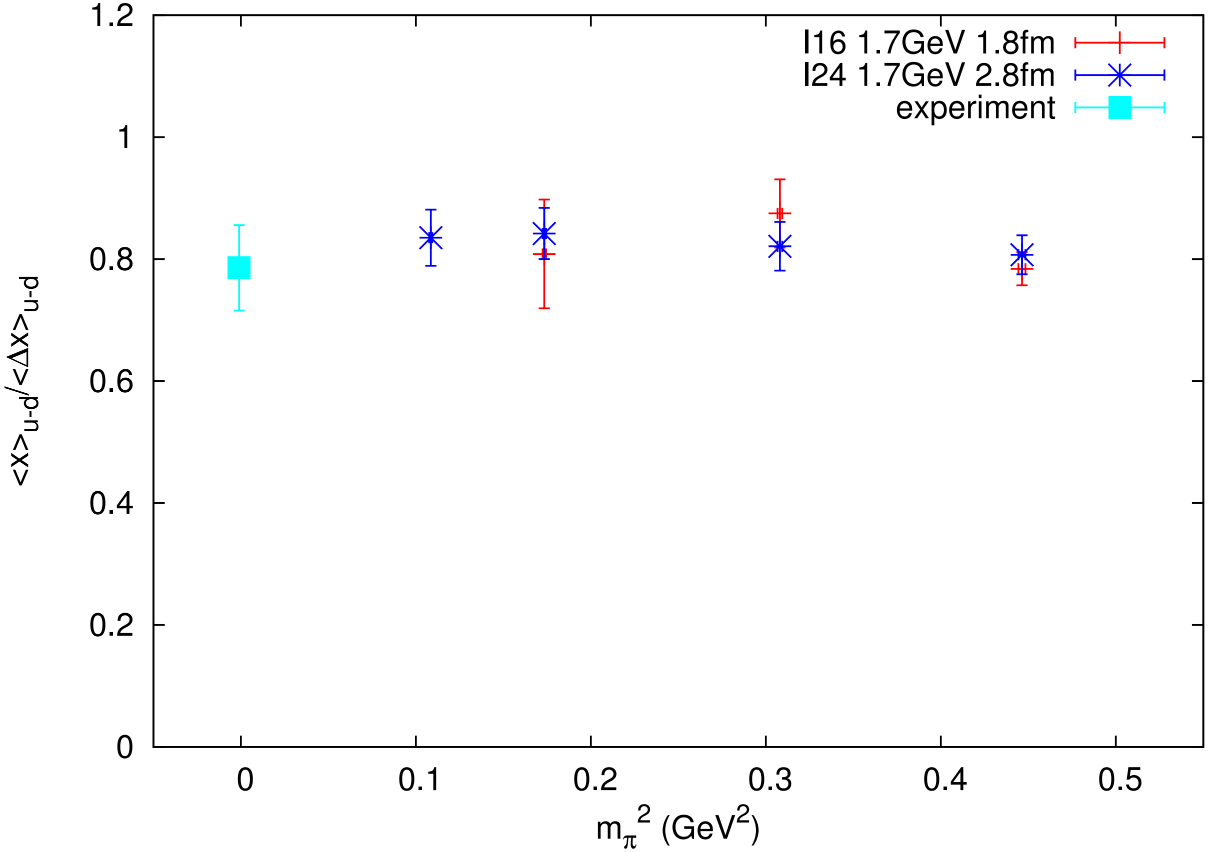
Ratio, $\langle x\rangle_{u-d}/\langle x\rangle_{\Delta u-\Delta d}$, of momentum and helicity fractions (naturally renormalized on the lattice),

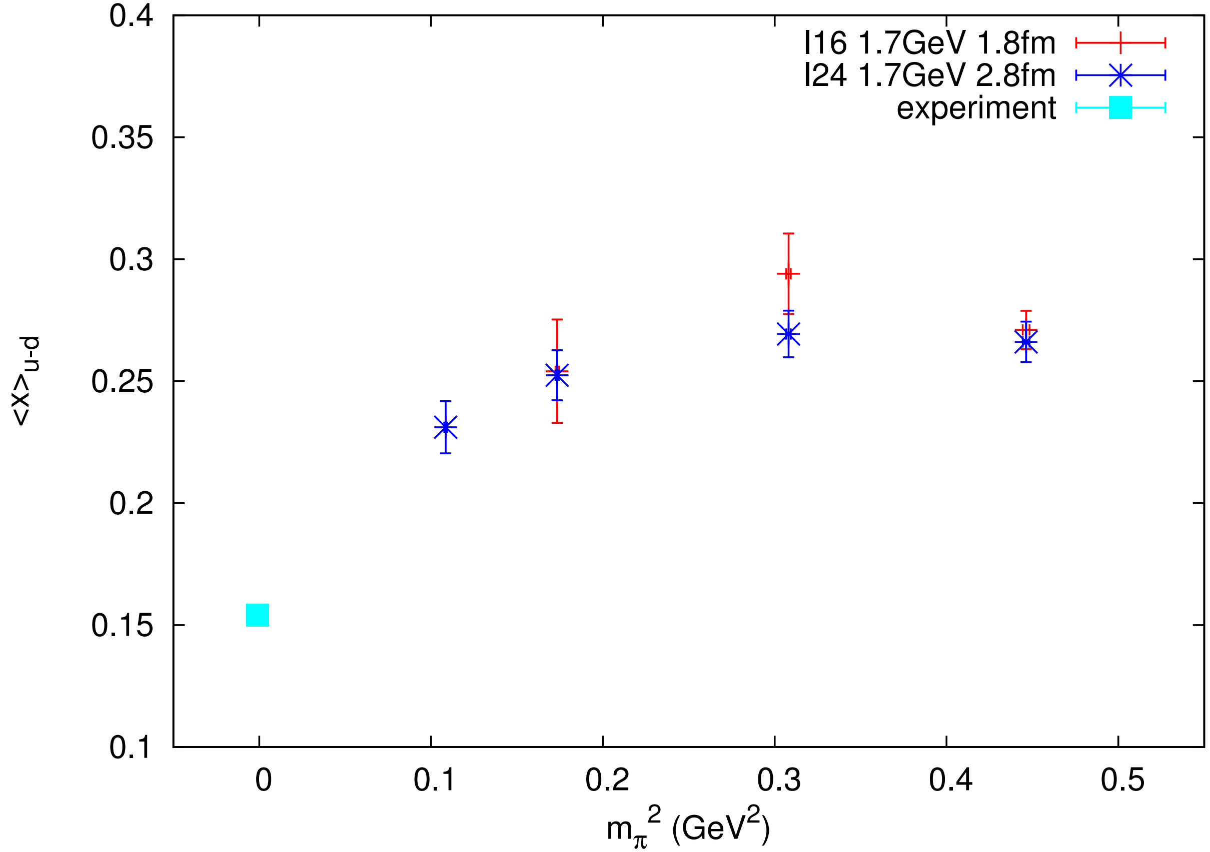


consistent with experiment, no discernible quark-mass dependence.
No finite-size effect seen, in contrast to g_A/g_V which is also naturally renormalized on the lattice.

RBC/UKQCD (2+1)-flavor, Iwasaki+DWF dynamical, $a^{-1} = 1.75(4)$ GeV, $m_{\rm res} = 0.00315(2)$, $m_{\rm strange} = 0.04$,

- $m_\pi = 0.67$, 0.56, 0.42 and 0.33 GeV; $m_N = 1.55$, 1.39, 1.22 and 1.15 GeV,

Momentum fraction, $\langle x \rangle_{u-d}$, with NPR, $Z^{\overline{\rm MS}}(2{\rm GeV}) = 1.15(4)$, plotted against m_π^2,



Absolute values have improved, trending to the experimental values, with NPR, $Z^{\overline{\rm MS}}(2{\rm GeV}) = 1.15(4)$.

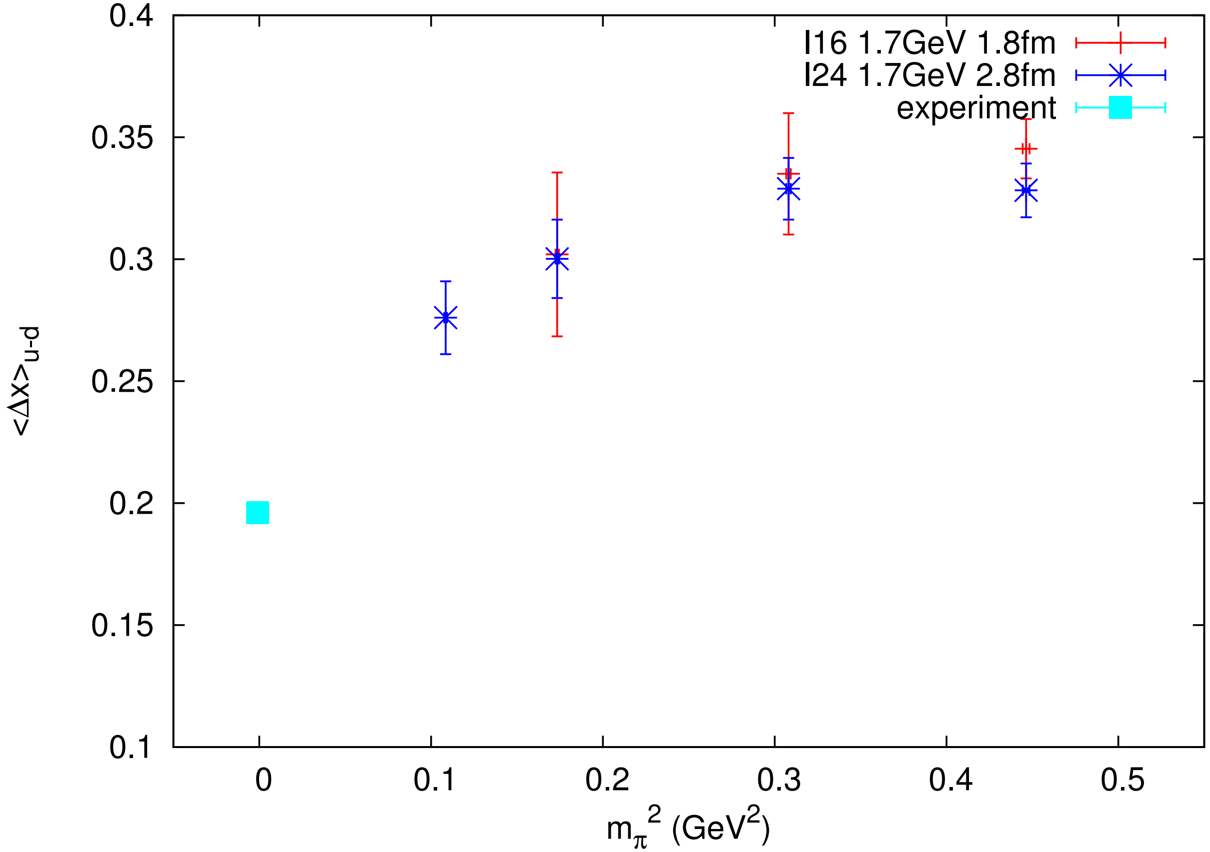
No finite size effect seen (16^3 (+) and 24^3 ($\times$) results agree): Likely physical light-quark effect.

A better understanding of quark mass dependence is necessary.

RBC/UKQCD (2+1)-flavor, Iwasaki+DWF dynamical, $a^{-1} = 1.75(4)$ GeV, $m_{\rm res} = 0.00315(2)$, $m_{\rm strange} = 0.04$,

- $m_\pi = 0.67, 0.56, 0.42$ and 0.33 GeV; $m_N = 1.55, 1.39, 1.22$ and 1.15 GeV,

Helicity fraction, $\langle x \rangle_{\Delta u - \Delta d}$, with NPR, $Z^{\overline{\rm MS}(2{\rm GeV})} = 1.15(3)$, plotted against m_π^2,



Absolute values have improved, trending to the experimental values, with NPR, $Z^{\overline{\rm MS}}(2{\rm GeV}) = 1.15(3)$.

No finite size effect seen (16^3 (+) and 24^3 (×) results agree): Likely physical light-quark effect.

A better understanding of quark mass dependence is necessary.

(2+1)-flavor dynamical lattice-QCD calculations of nucleon structure so far ($m_\pi \sim$300 MeV) give

- much too small radii for vector-current form factors,
  - The vector-current form factors are confirmed by LHP at a higher cut off, $\sim$ 2.3 GeV, using another set of RBC+UKQCD (2+1)-flavor dynamical DWF ensembles.
- while axial-current form factors seem to overflow,
- but structure function moments may be starting to behave.

Lighter quark/pion mass should help: requires large volume, $m_\pi L \gg 4$.

- Light $m_\pi \sim$ light quark mass $\rightarrow$ small $m_{\rm res}$.
- Large $L \sim$ coarse lattice? $\sim$ more topological dislocations? $\rightarrow$ not so small $m_{\rm res}$?

Can we achieve sufficiently small $m_{\rm res}$ with reasonable topological distribution?

Now RBC and UKQCD collaborations are jointly generating new (2+1)-flavor DWF ensembles

- with Iwasaki and dislocation-suppressing-determinant-ratio (DSDR) gauge action, $\beta = 1.75$,
- and DWF fermion action, $L_s = 32$ and $M_5 = 1.8$, with $m_{\rm strange} = 0.045$, $m_{\rm ud} = 0.0042$ and 0.001,

using FNAL ALCF, a BG/P facility.

We have reasonable topology distribution while maintaining small residual mass, $m_{\rm res}a \sim 0.002$:

- lattice scale from Ω^-: $a^{-1} = 1.371(10)$ GeV,
- $m_\pi = 0.1816(8)$ and $0.1267(8)$, or $\sim$ 250 and 170 MeV,
- $32^3 \times 64$ volume is about 4.6 fm across in space, 9.2 fm in time.

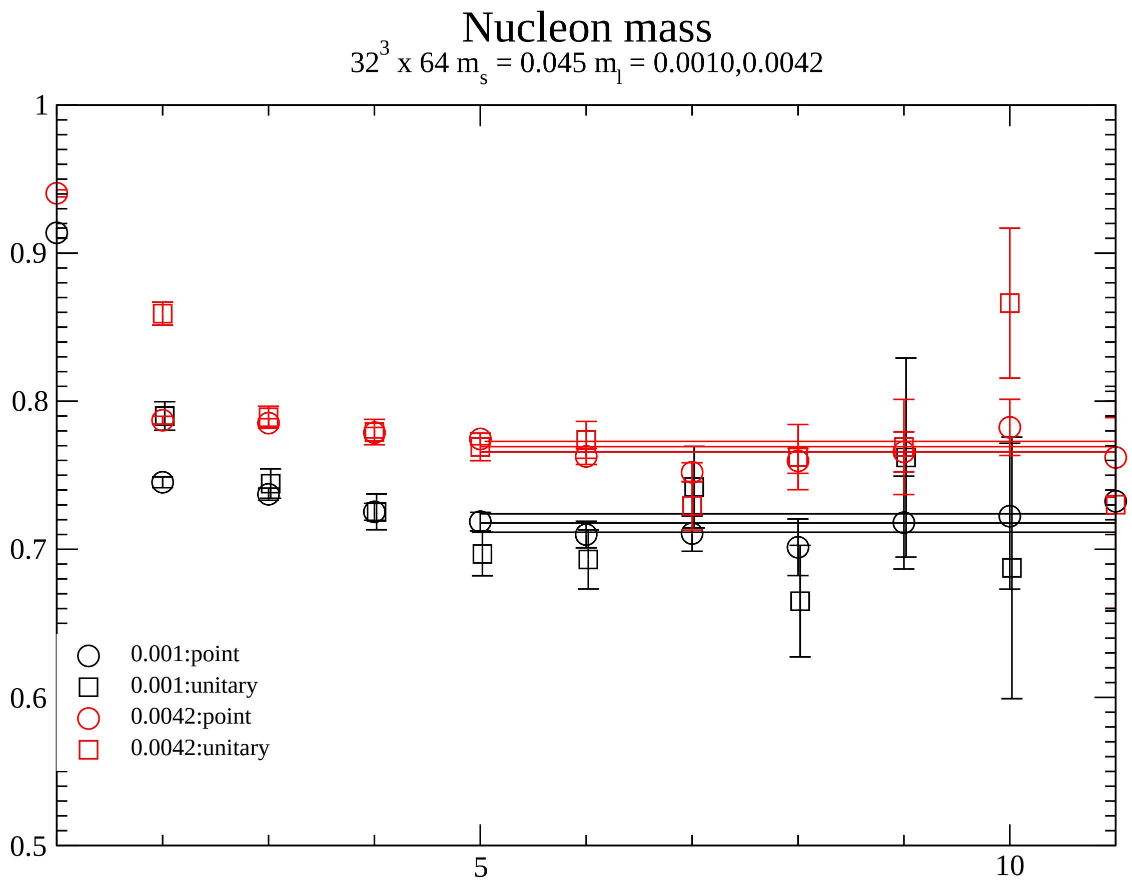
NLO chiral perturbation seems making sense.

We started nucleon structure calculations:

- finished tuing Gaussian smearing, width 6 favored over 4.
- sink separation at 9, four source positions per configuarion so far,
- 608–1920/8 for 250-MeV, 508–1412/8 for 170-MeV so far analyzed for 3pt,

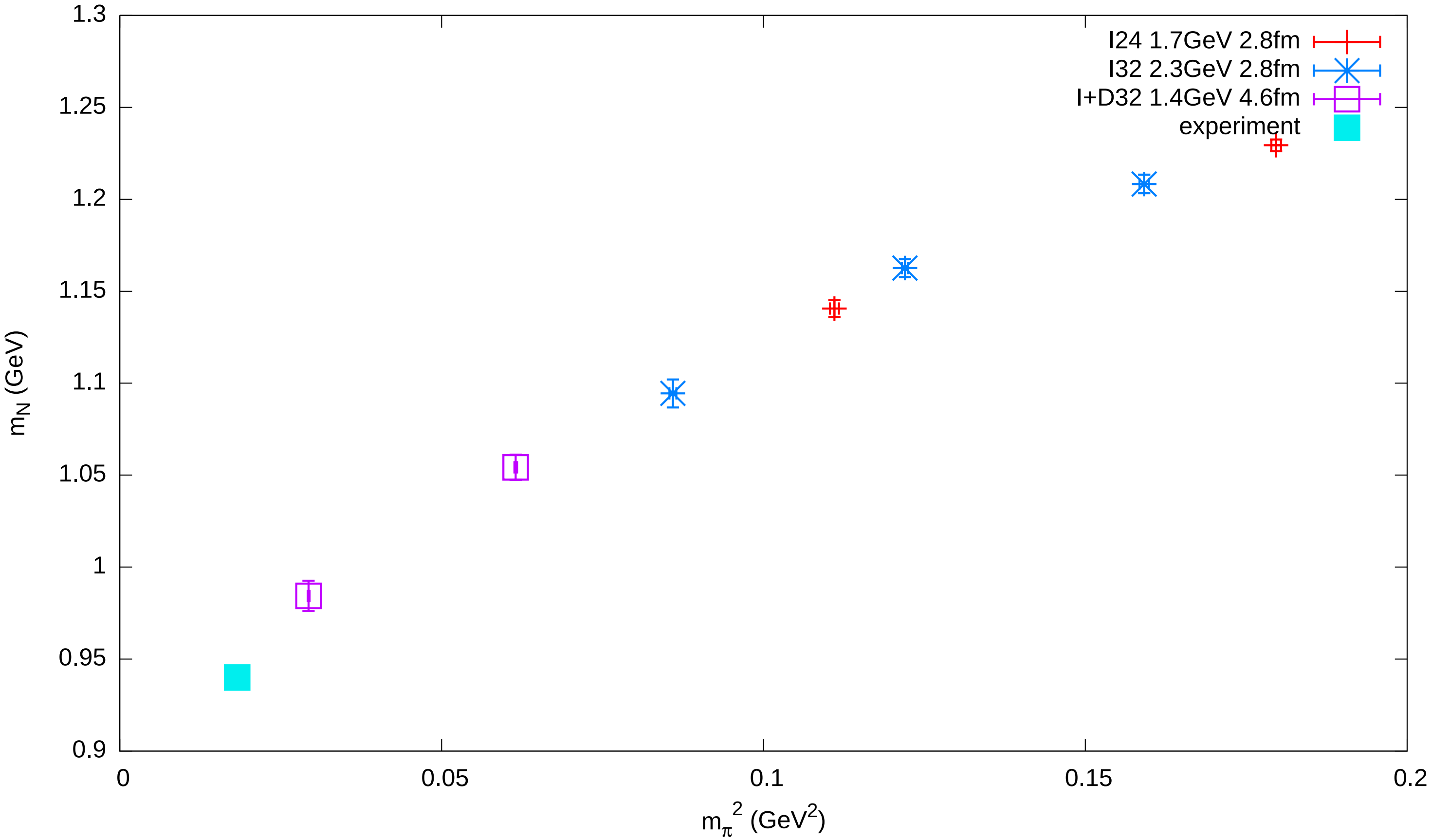
thanks to RICC/RIKEN and Teragrid/XSEDE clusters.

RBC/UKQCD (2+1)-flavor, ID+DWF dynamical, $a^{-1} = 1.371(10)$ GeV,



$m_N = 0.718(6)$ or ~ 0.98 GeV for $m_\pi \sim 170$MeV, and $m_N = 0.769(5)$ or ~ 1.05 GeV for $m_\pi \sim 250$MeV.

Nucleon mass: RBC/UKQCD (2+1)-flavor, ID+DWF ensembles are being analyzed for nucleon physics.

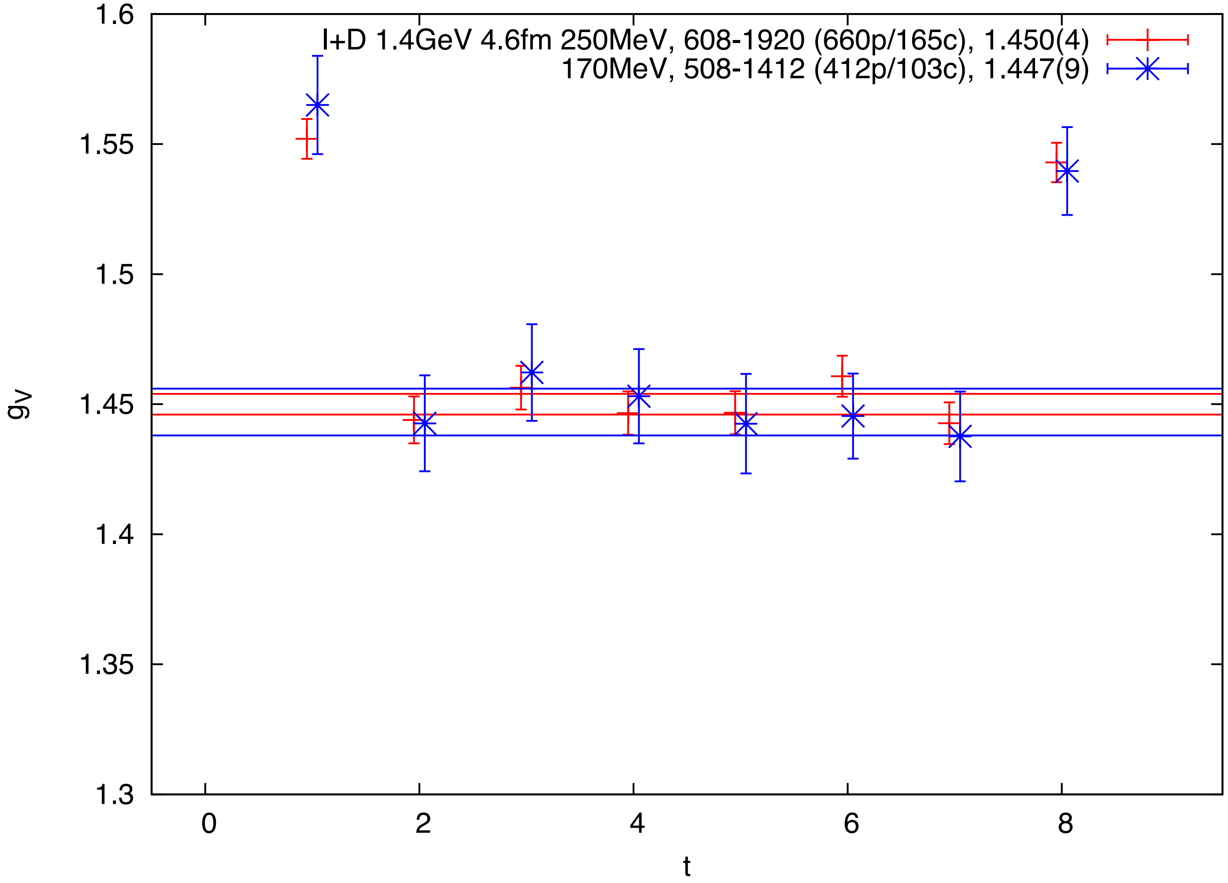


with $a^{-1} = 1.371(10)$ GeV, $(\sim 4.6\text{fm})^3$ spatial volume.

Closer to physical mass, $m_\pi = 170$ and 250 MeV, $m_N < 1.0$ GeV,

.

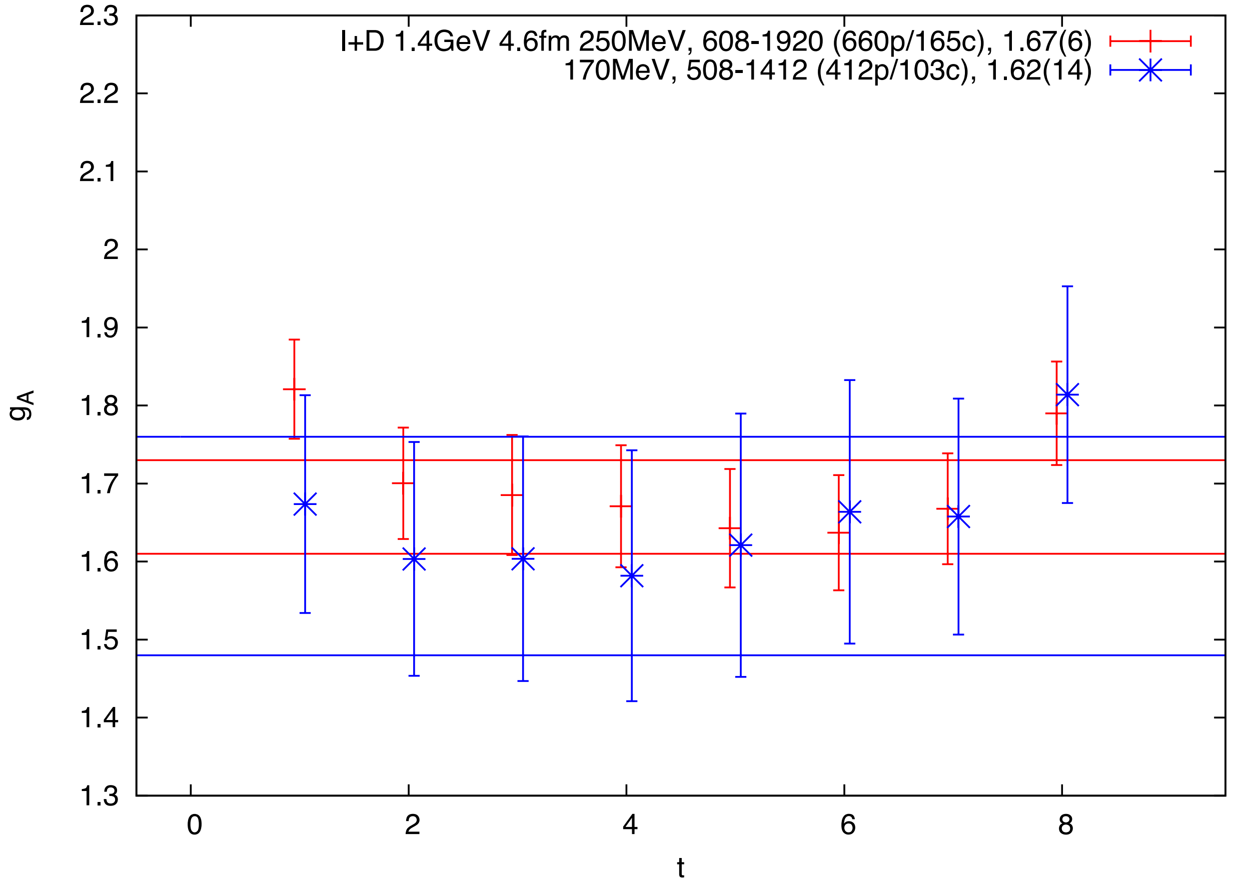
Nucleon isovector 3-pt functions are being obtained: 608-1920 for 250-MeV, 508-1412 for 170-MeV.



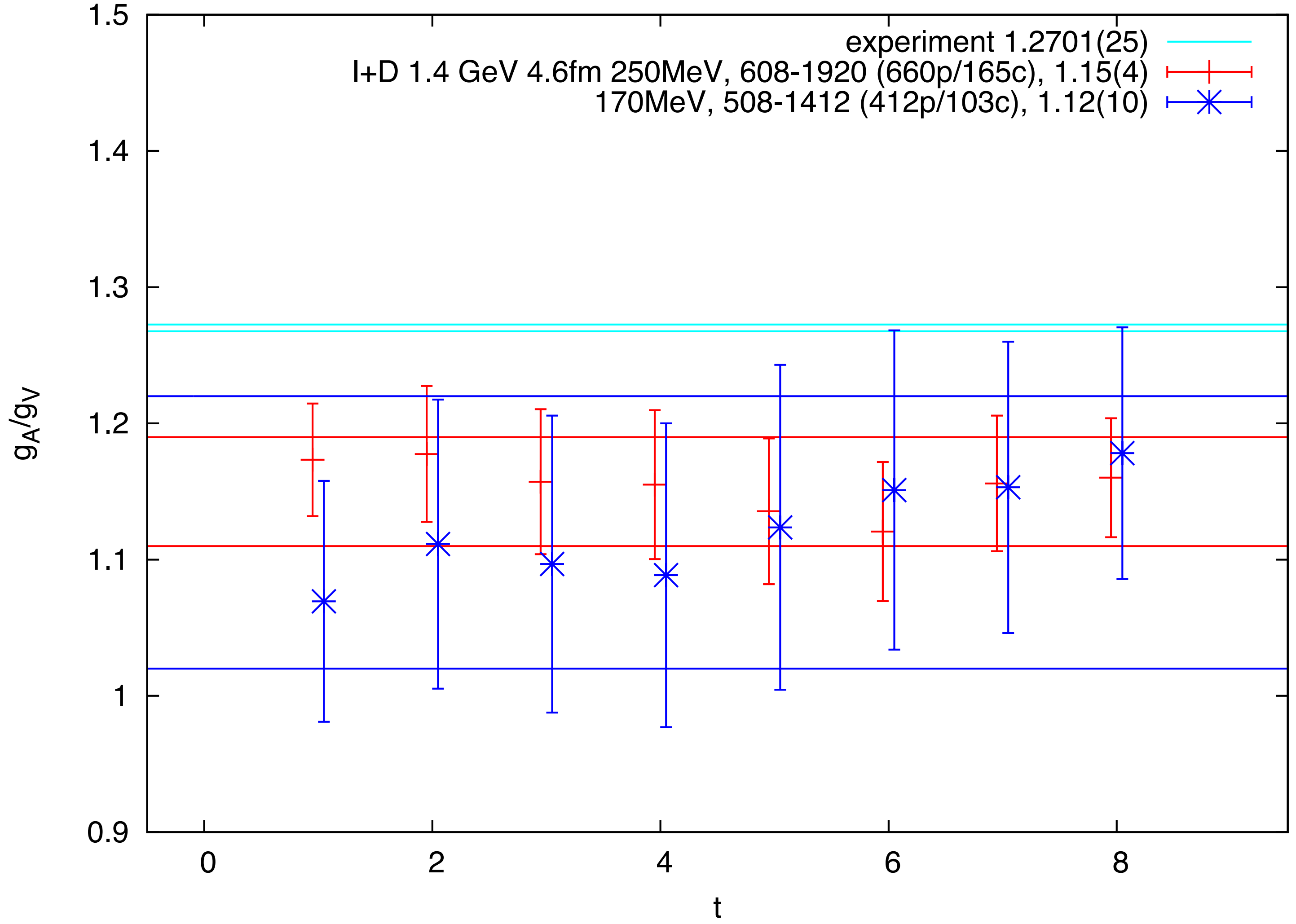
Local-current isovector vector charge, $g_V = 1.450(4)$ or $1.447(9)$, is obtained, corresponding to $Z_V = 0.692(7)$,

- in good agreement with $Z_V = 0.673(8)$ and $Z_A = 0.6878(3)$ obtained in the meson sector,
- yet again proving good chiral and flavor symmetries up to $O(a^2)$.

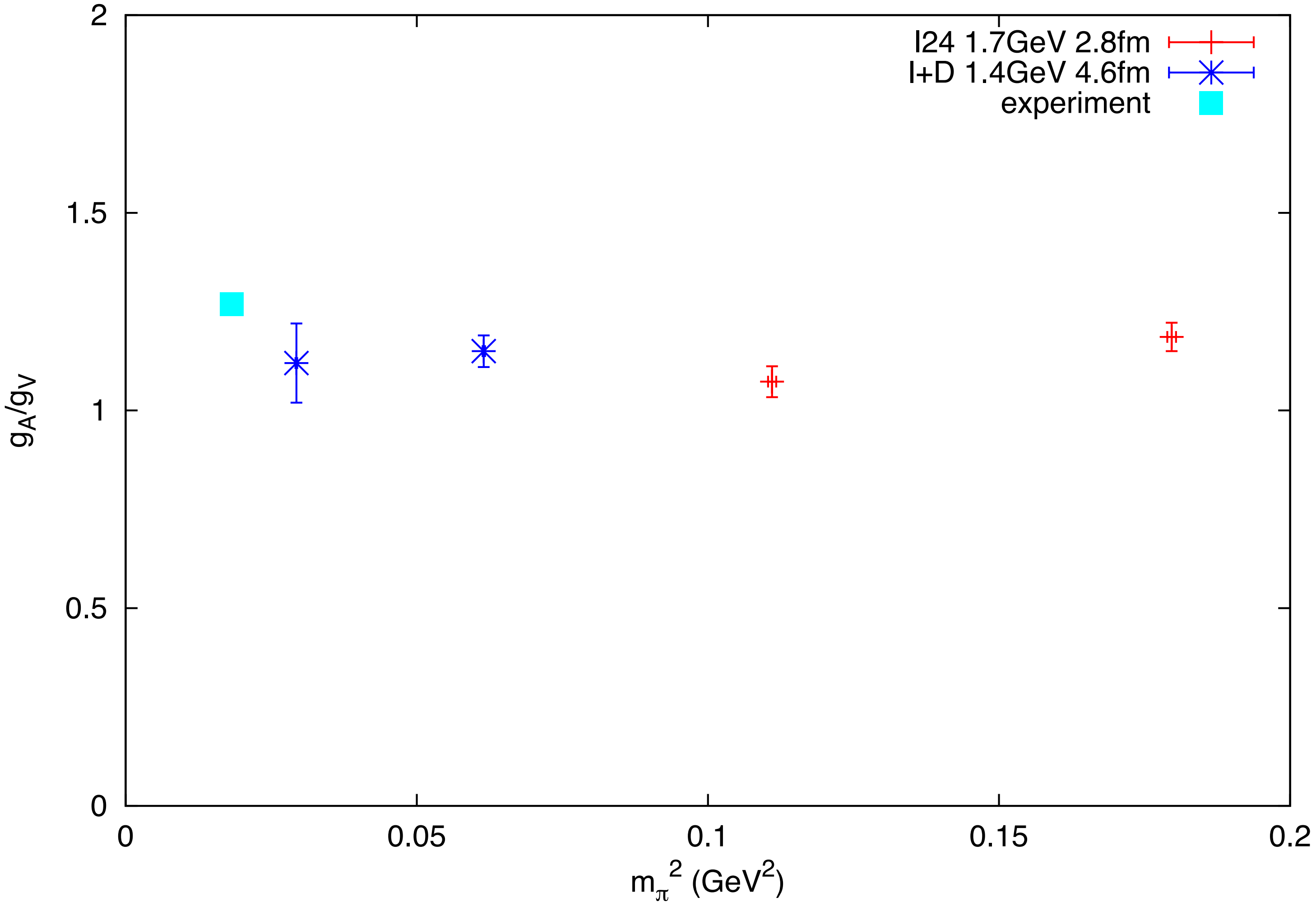
Axialvector current: Noisier than vector current, as expected,



g_A/g_V, ratio of isovector axial and vector charges, is less noisy, again as expected,

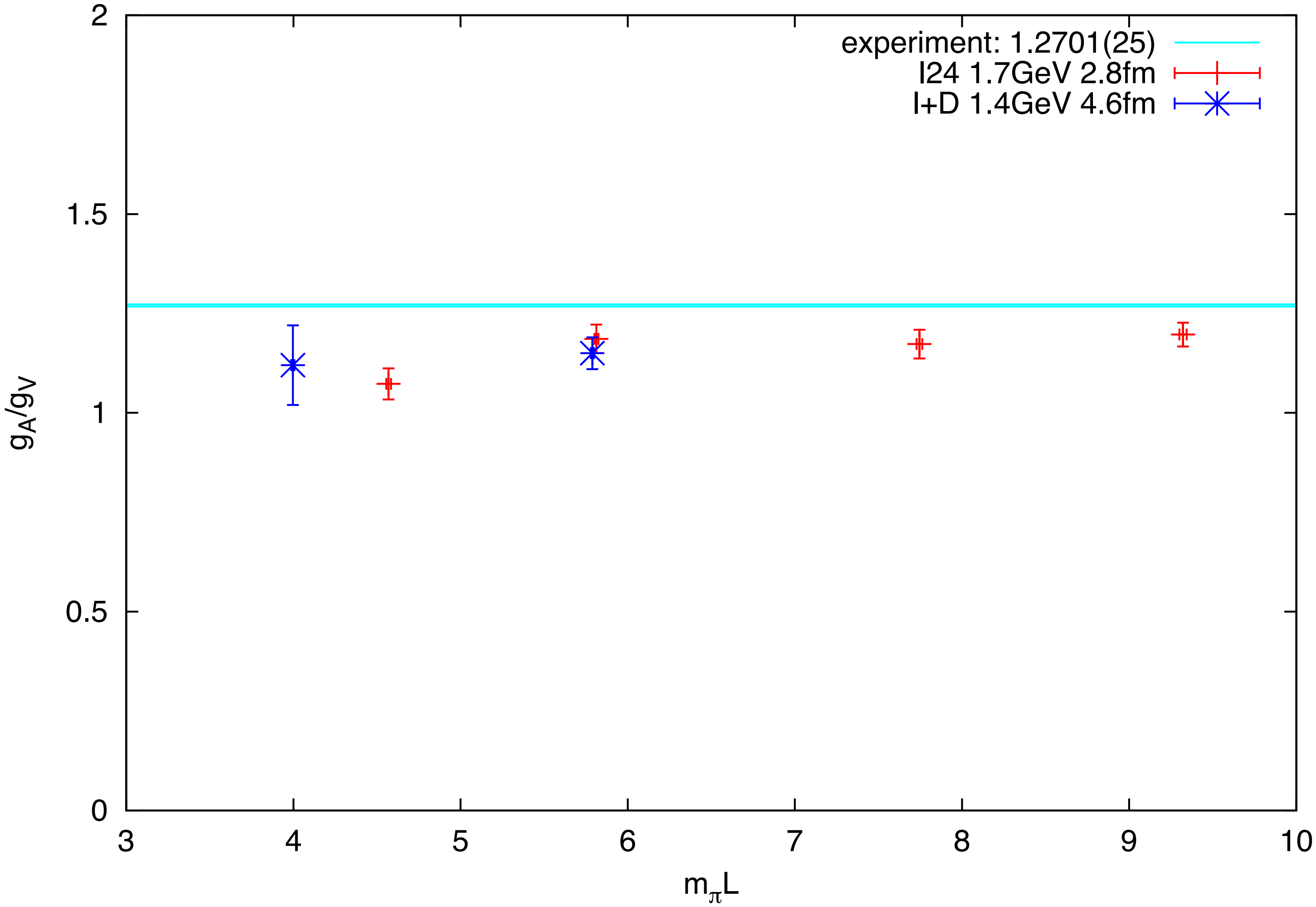


g_A/g_V: seems to stay away from the experiment as we set the pion mass lighter.



Not monotonic: appears to be a finite-size effect.

g_A/g_V: appears to show finite-size effect that is consistent with scaling in $m_\pi L$.



Results from two ensembles, 1.19(4) from I24 and 1.15(5) from ID, agree with each other, despite very much different m_π that significantly alter mass spectrum. There does not seem excited-state contamination above our statistics.

Results from two ensembles, I24 and ID32, which differ in

- quark/pion mass, m_π of 420 MeV and 250 MeV,
- spatial volume, L of 2.8 fm and 4.6 fm,
- lattice cut off, a^{-1} of 1.7 GeV and 1.4 GeV,
- gauge actions,

that should give different source, $A_0' e^{-E_0 t}|0\rangle + A_1' e^{-E_1 t}|1\rangle + \ldots$, agree well in g_A/g_V when $m_\pi L$ agree:

Do the differences magically conspire, or do we see scaling in $m_\pi L$?

Yet we liked to improve the statistical significance:

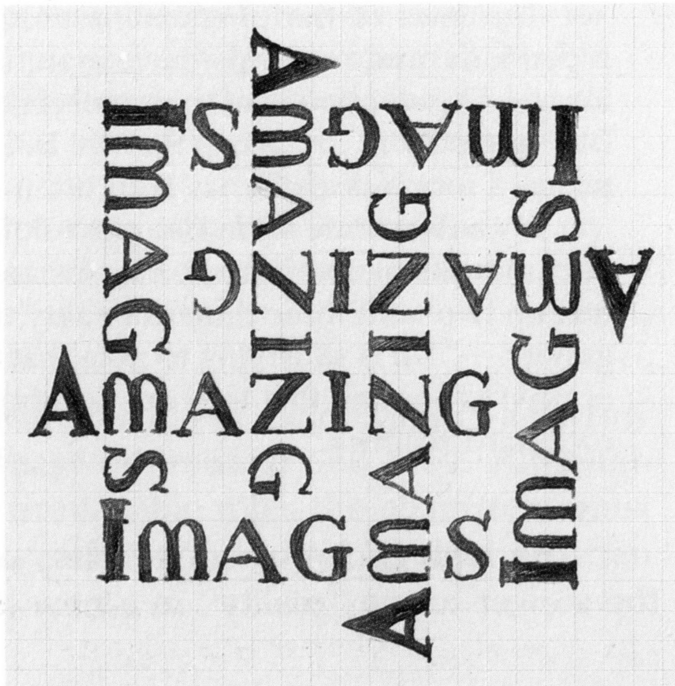
Not so trivial a task,

as the results took a few years using US and Japanese national clusters, XSEDE and RICC.

A new statistical technique, "AMA[6]," offer $\times$10–20 acceleration: by allowing

- cruder,
- but cheaper,

independent statistical sampling at much higher frequency, by taking advantage of point-group symmetries of the lattice to organize many such cruder but independent and equivalent measurements:



$$\langle O \rangle_{\rm AMA} = \frac{1}{N_{\rm sloppy}} \sum_{s}^{N_{\rm sloppy}} \langle O \rangle_{\rm sloppy}^{s} + \frac{1}{N_{\rm accurate}} \sum_{a}^{N_{\rm accurate}} \left(\langle O \rangle_{\rm accurate}^{a} - \langle O \rangle_{\rm sloppy}^{a} \right)$$

[6]T. Blum, T. Izubuchi and E. Shintani, arXiv:1208.4349; PoS Lattice 2012, 262.

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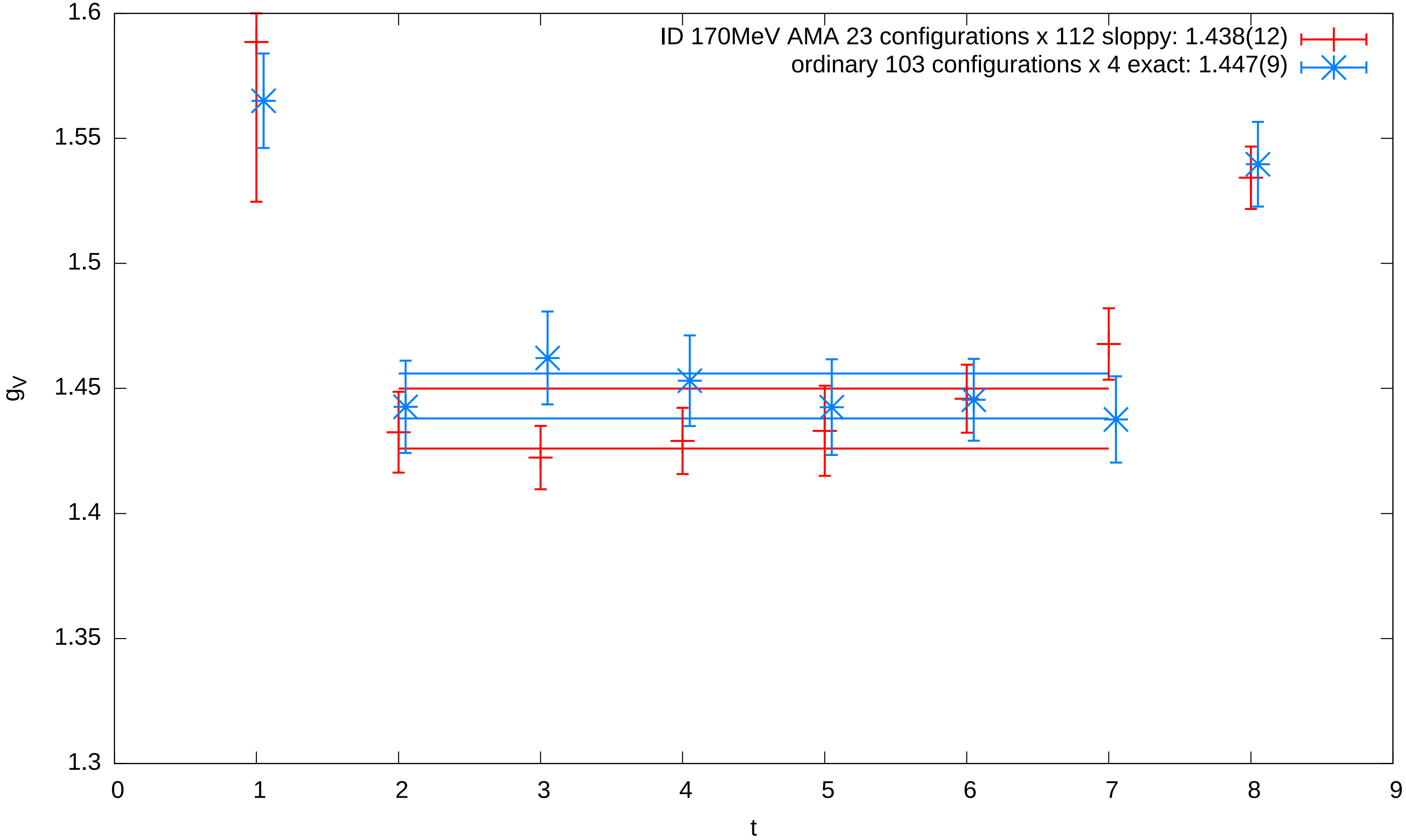

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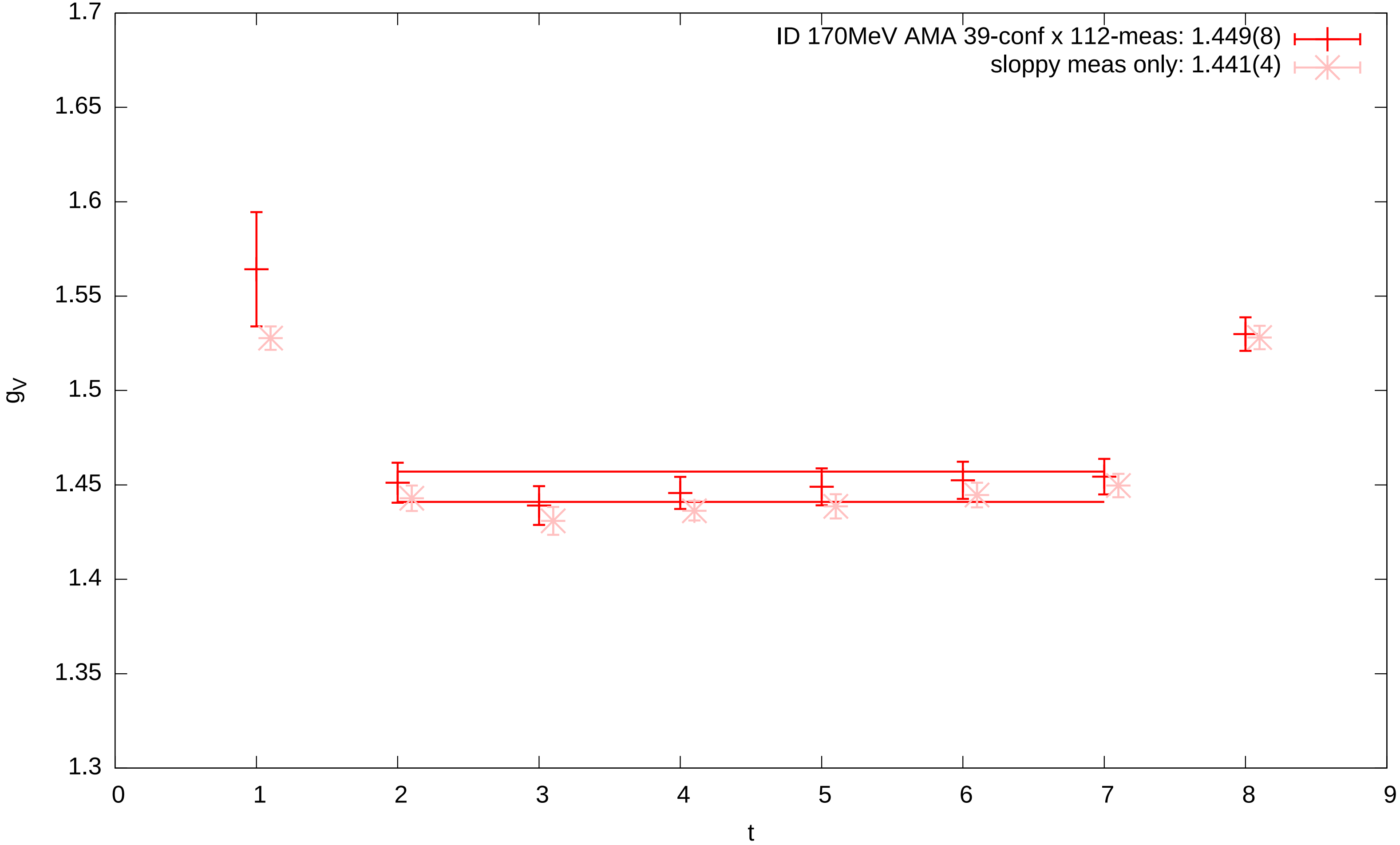

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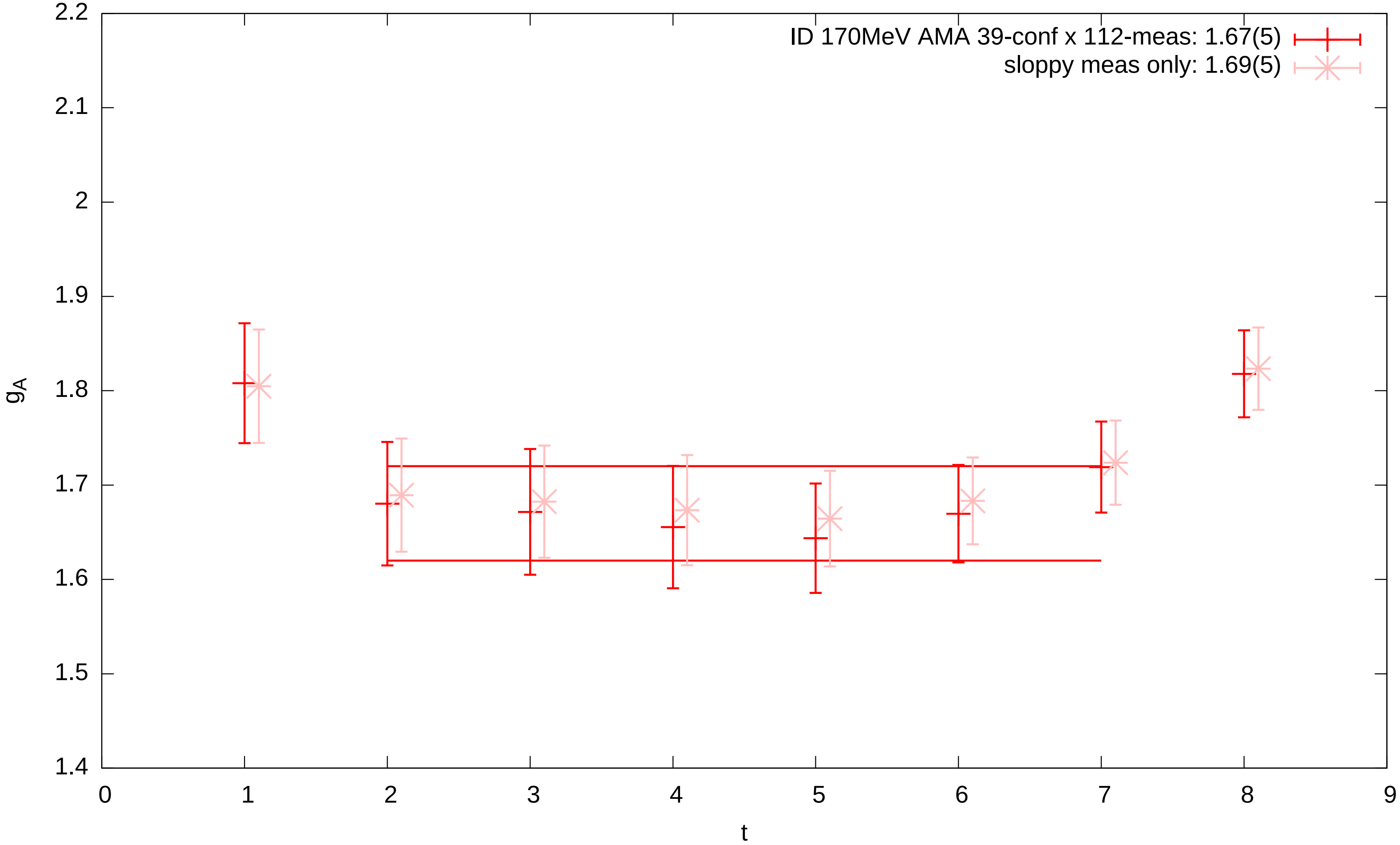

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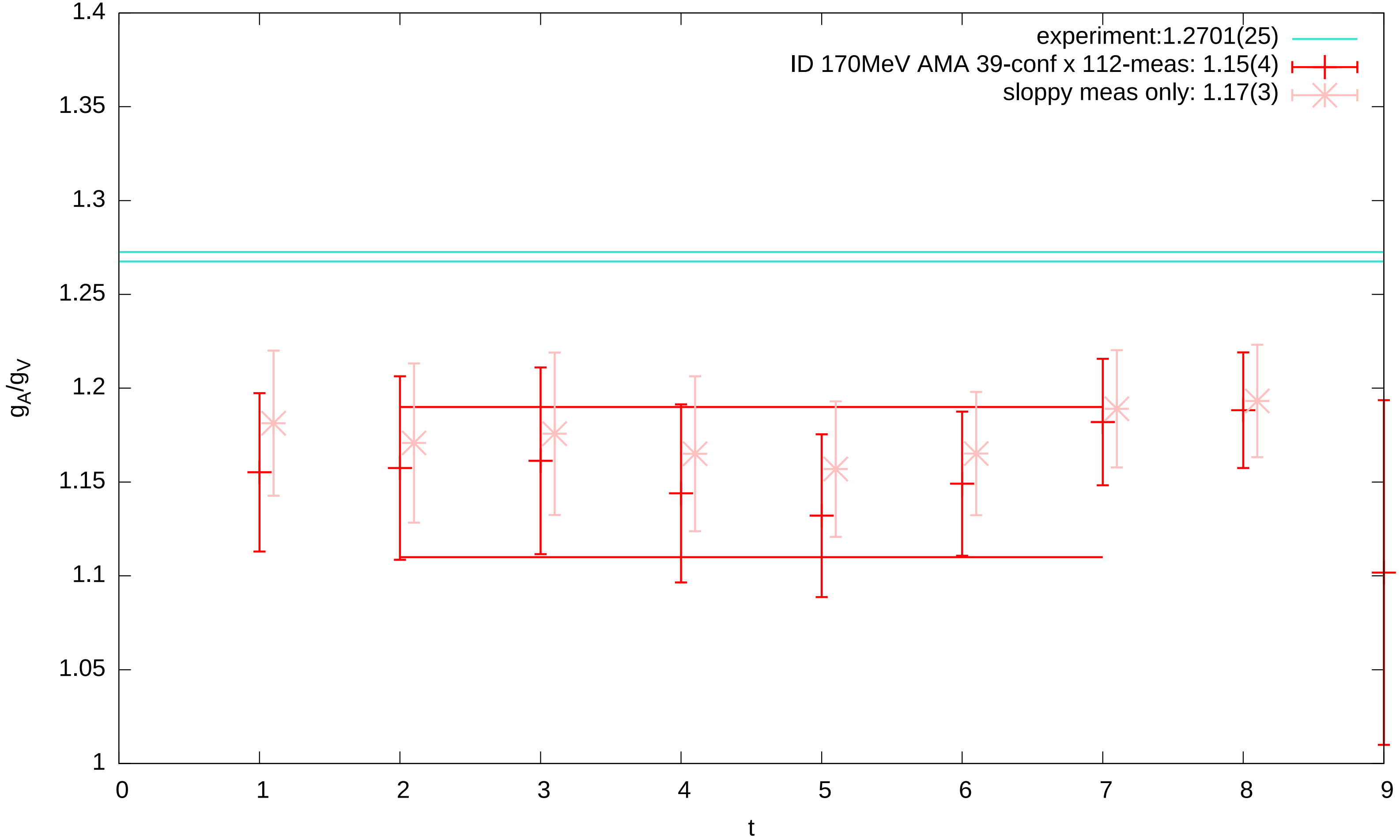

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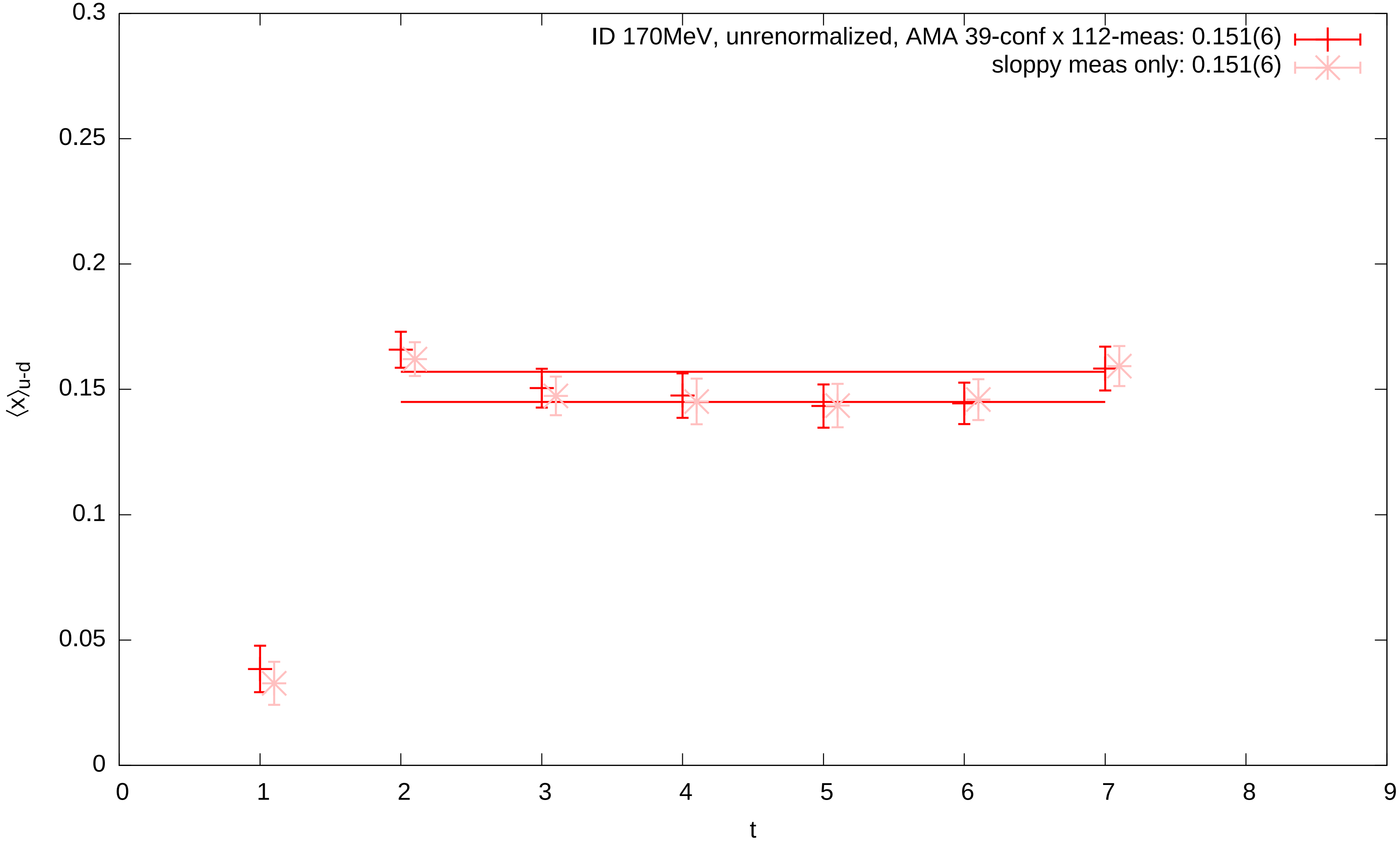

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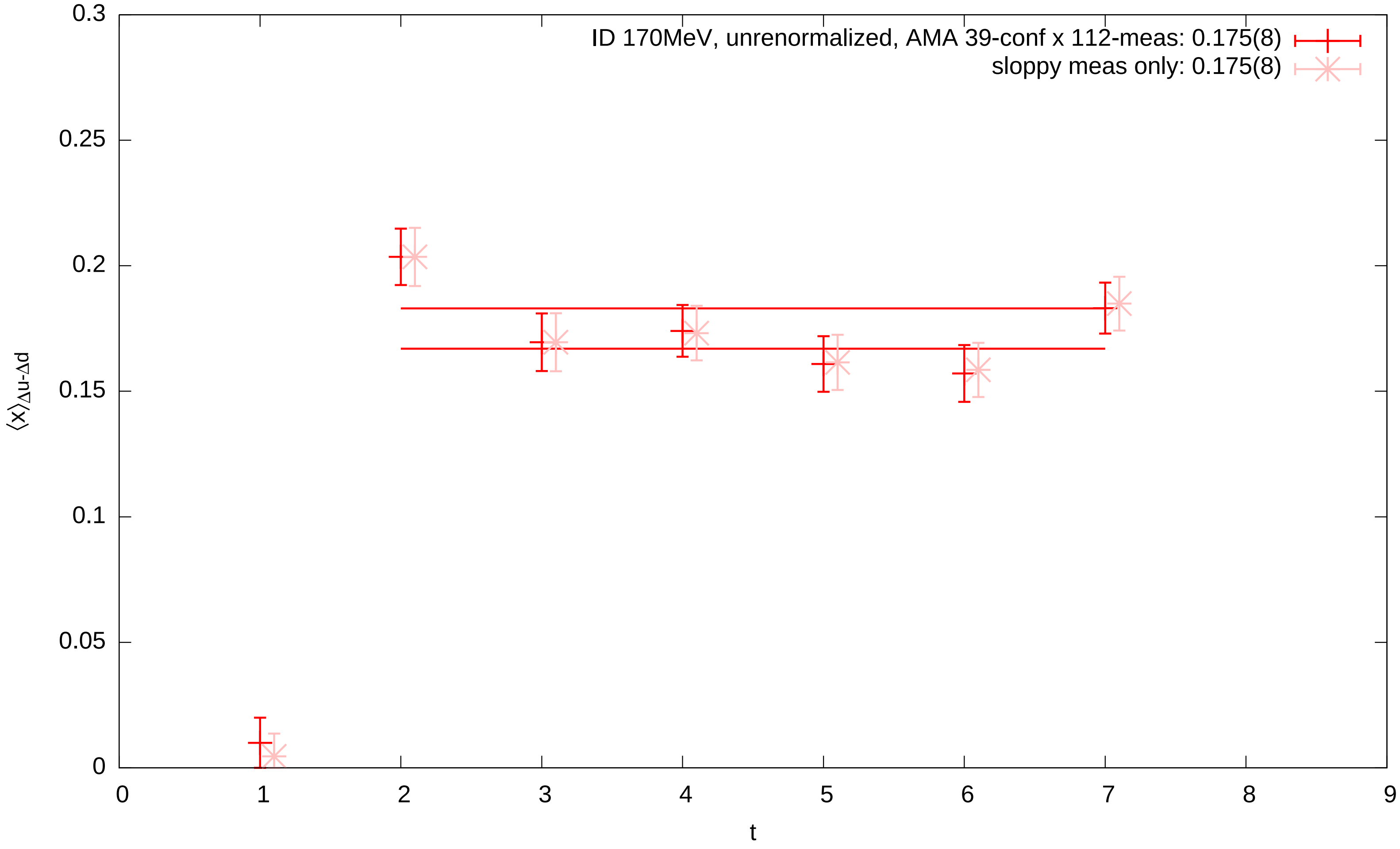

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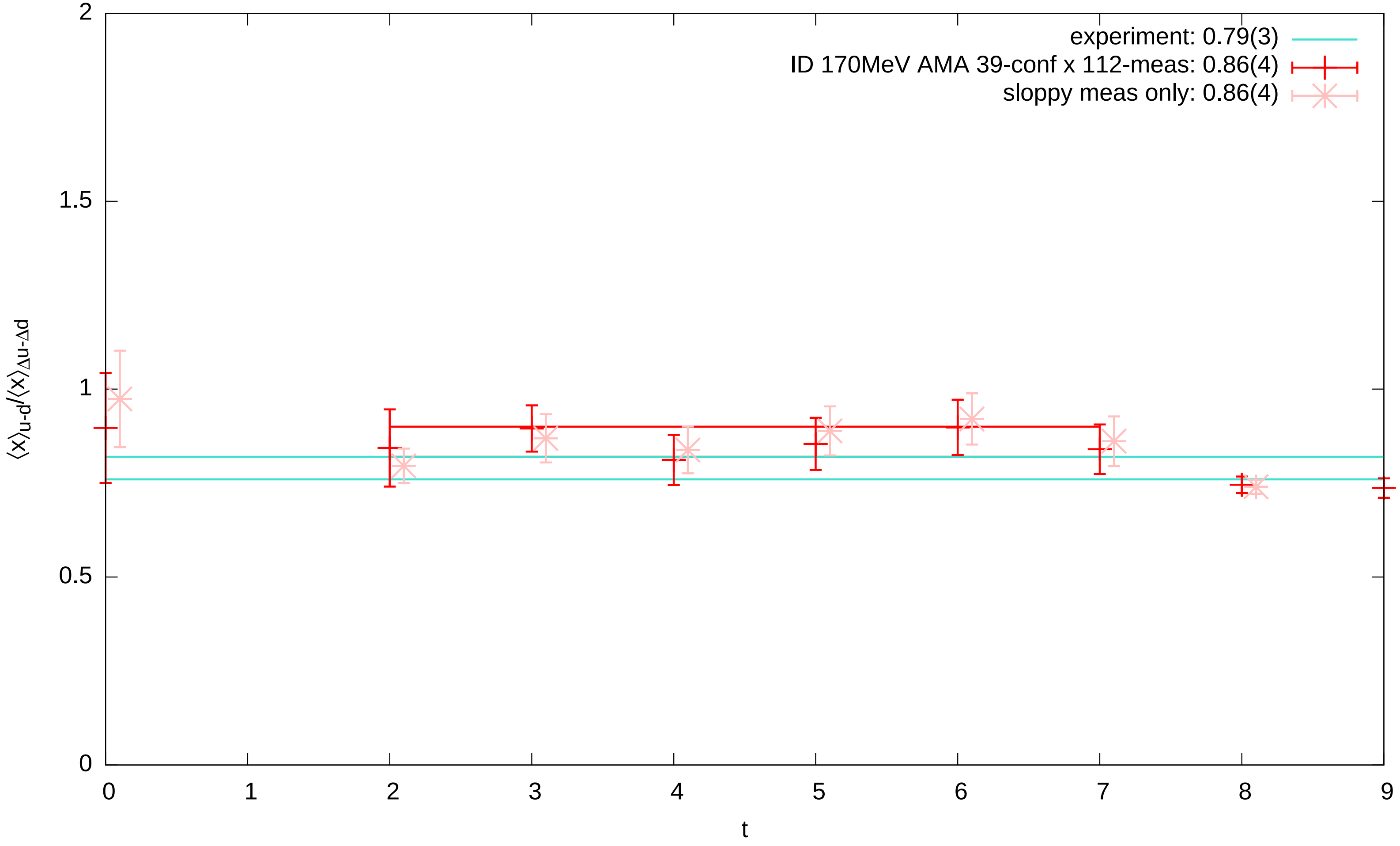
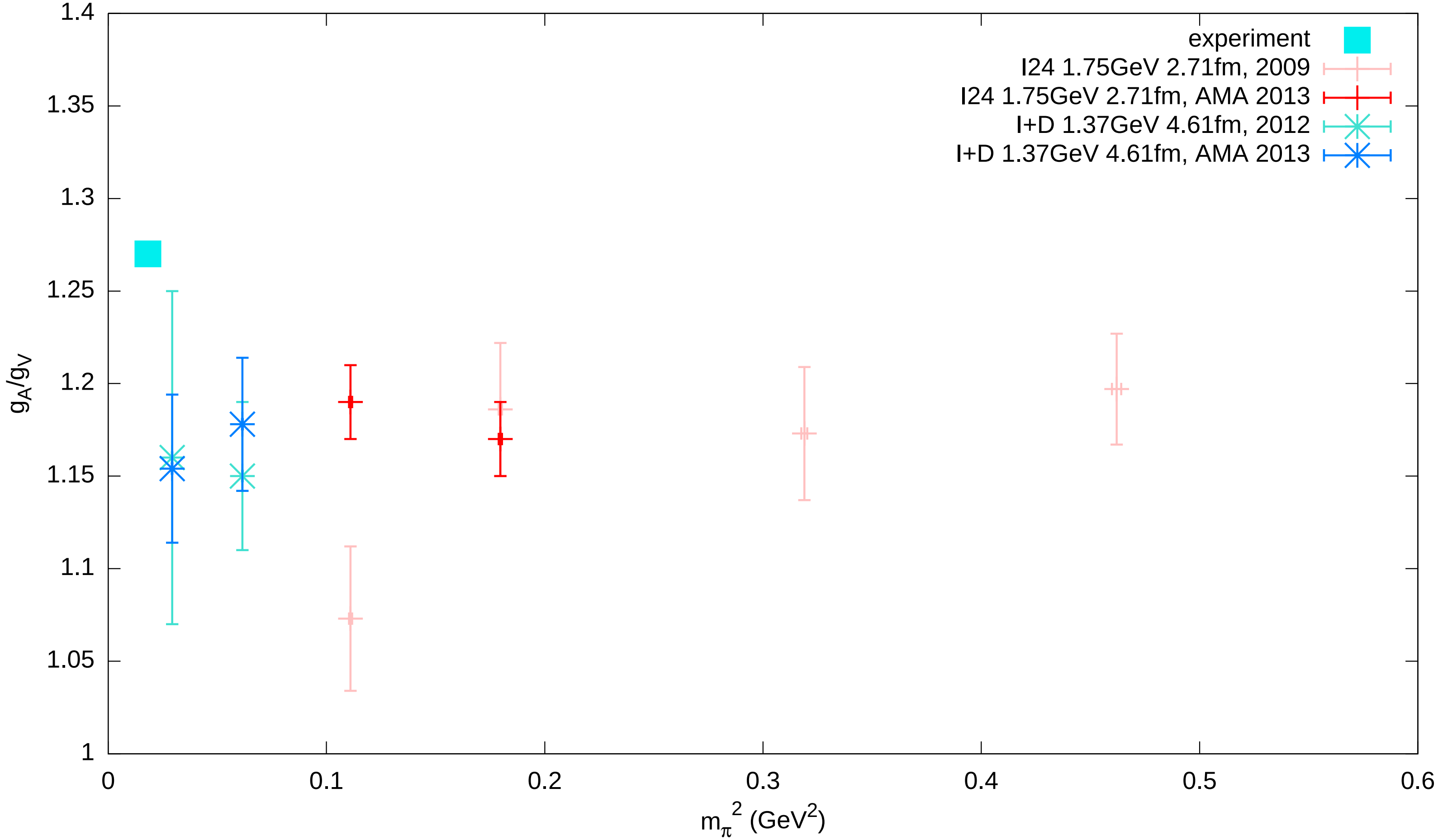
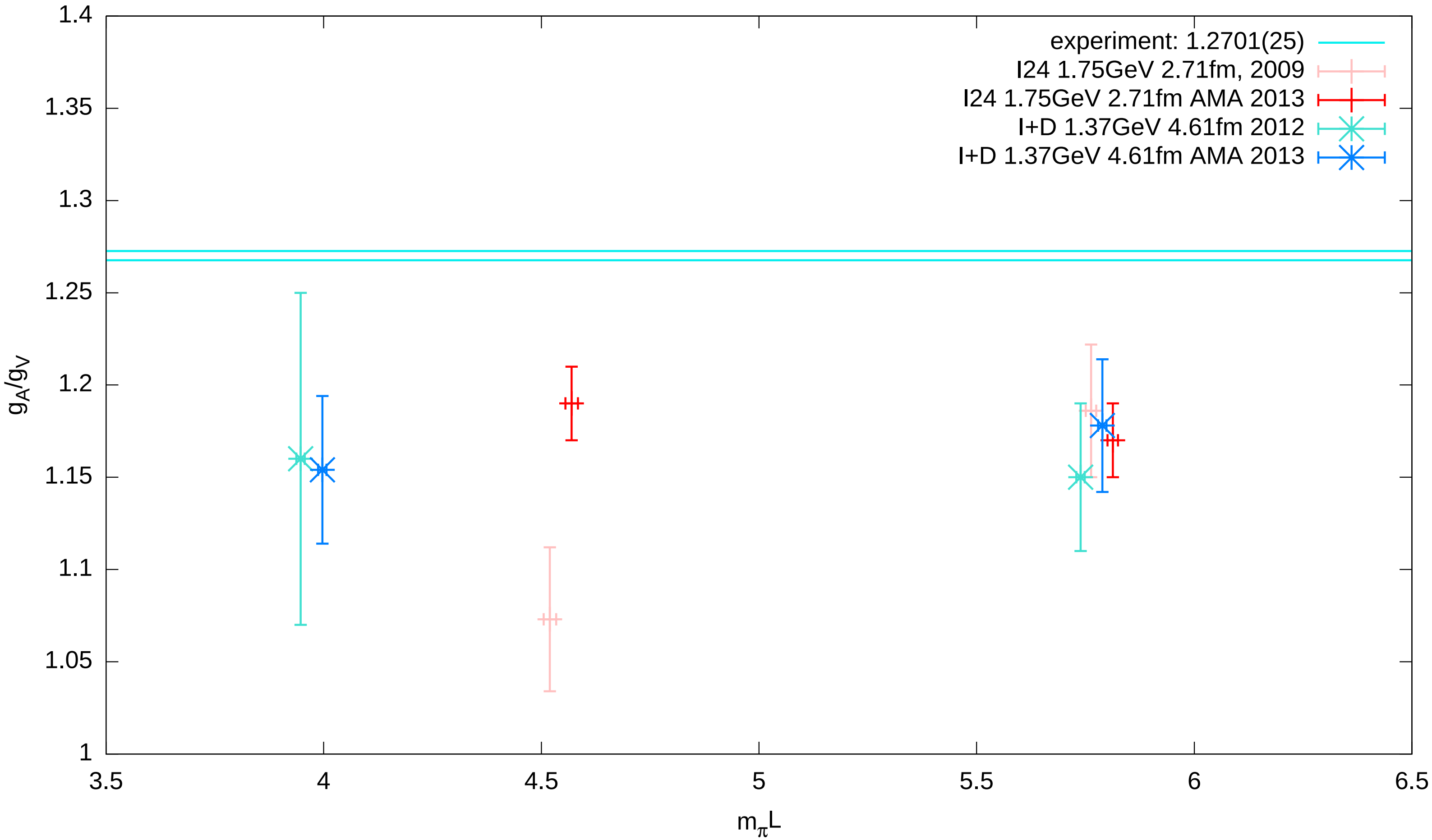


With AMA and other statistical improvements, g_A/g_V vs m_π^2 now looks like the following:



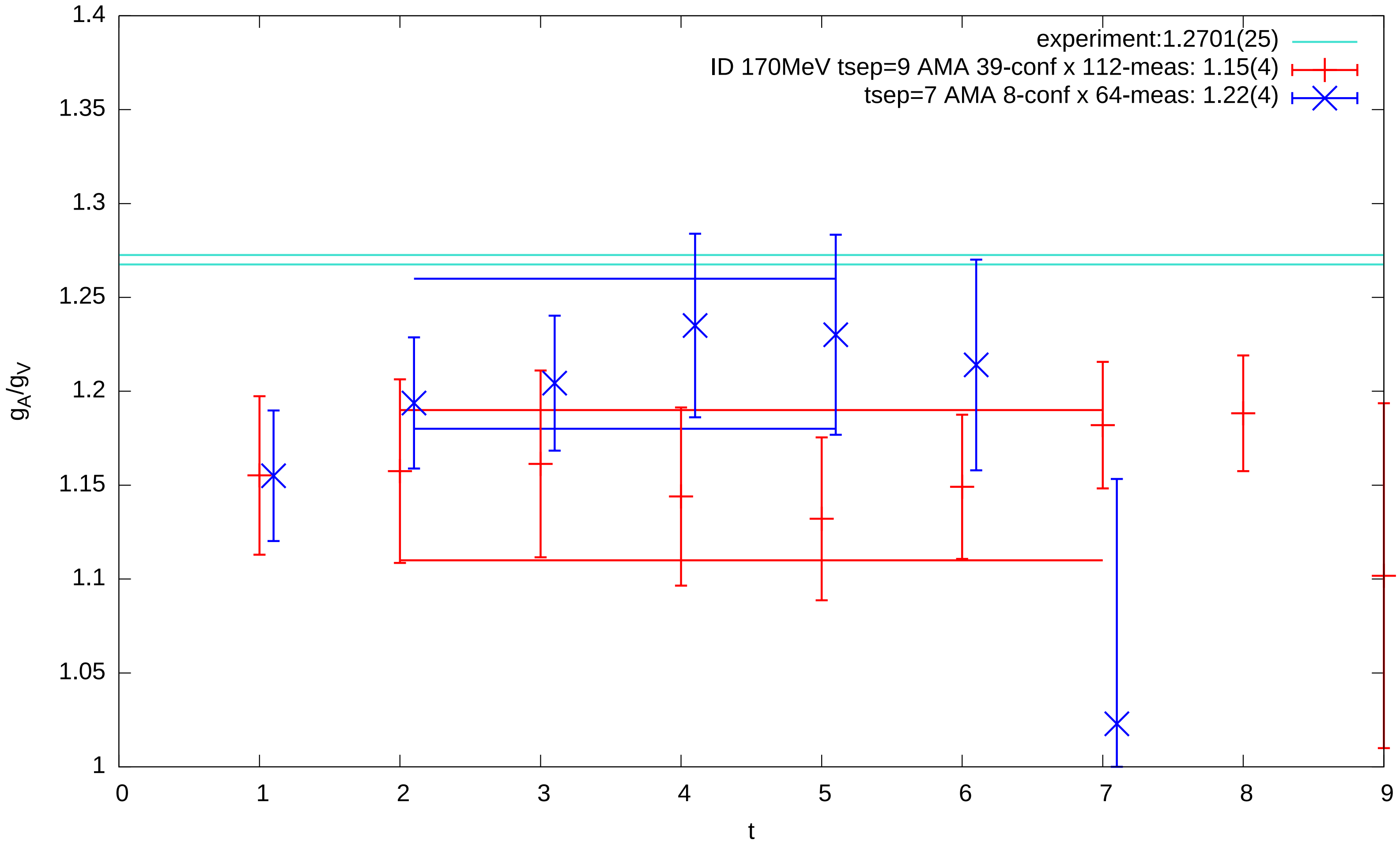
Moves away from the experiment as m_π approaches the experimental value.

With AMA and other statistical improvements, g_A/g_V agreement at $m_\pi L = 5.8$ is more significant: 1.17(2) and 1.15(4)



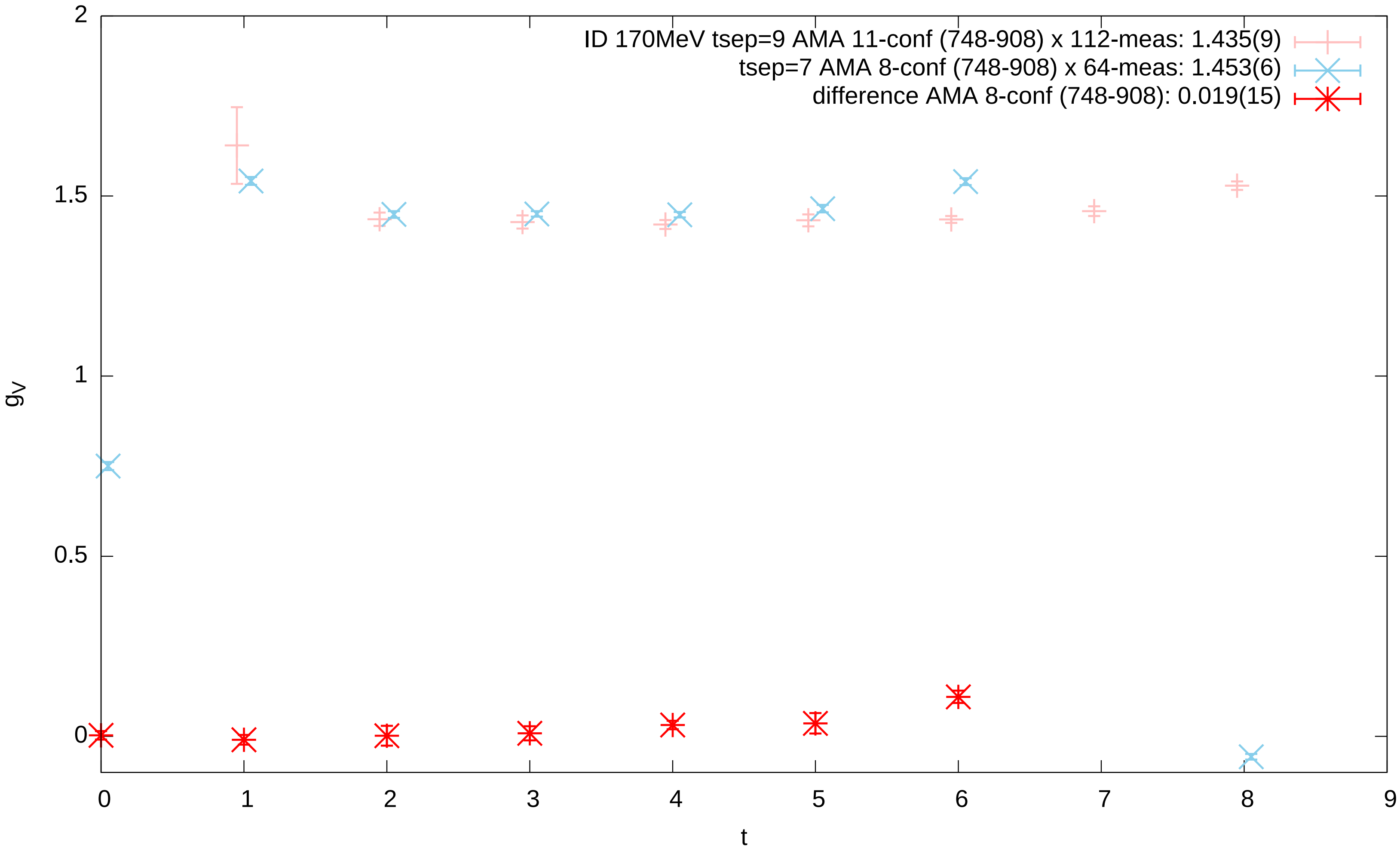
About 10-% deficit confirmed?

Also with new AMA calculations, this deficit in g_A/g_V seem less likely from excited states:



Results at shorter $t_{\text{sink}} - t_{\text{source}} = 7$ should suffer more excited-state:
though statistically not significant, they seem to give (systematically) higher g_A/g_V.
$t_{\text{sep}} = 9$ result is lower than experiment even when we consider excited-state contamination.

In fact, no excited-state contamination is seen in any of our 170-MeV calculations:



When compared with the same configurations, the difference is always consistent with 0.
$A_1\langle 1|O|0\rangle \sim 0$ for any observable we look at: A_1 is negligible for these small $\langle 1|O|0\rangle$.

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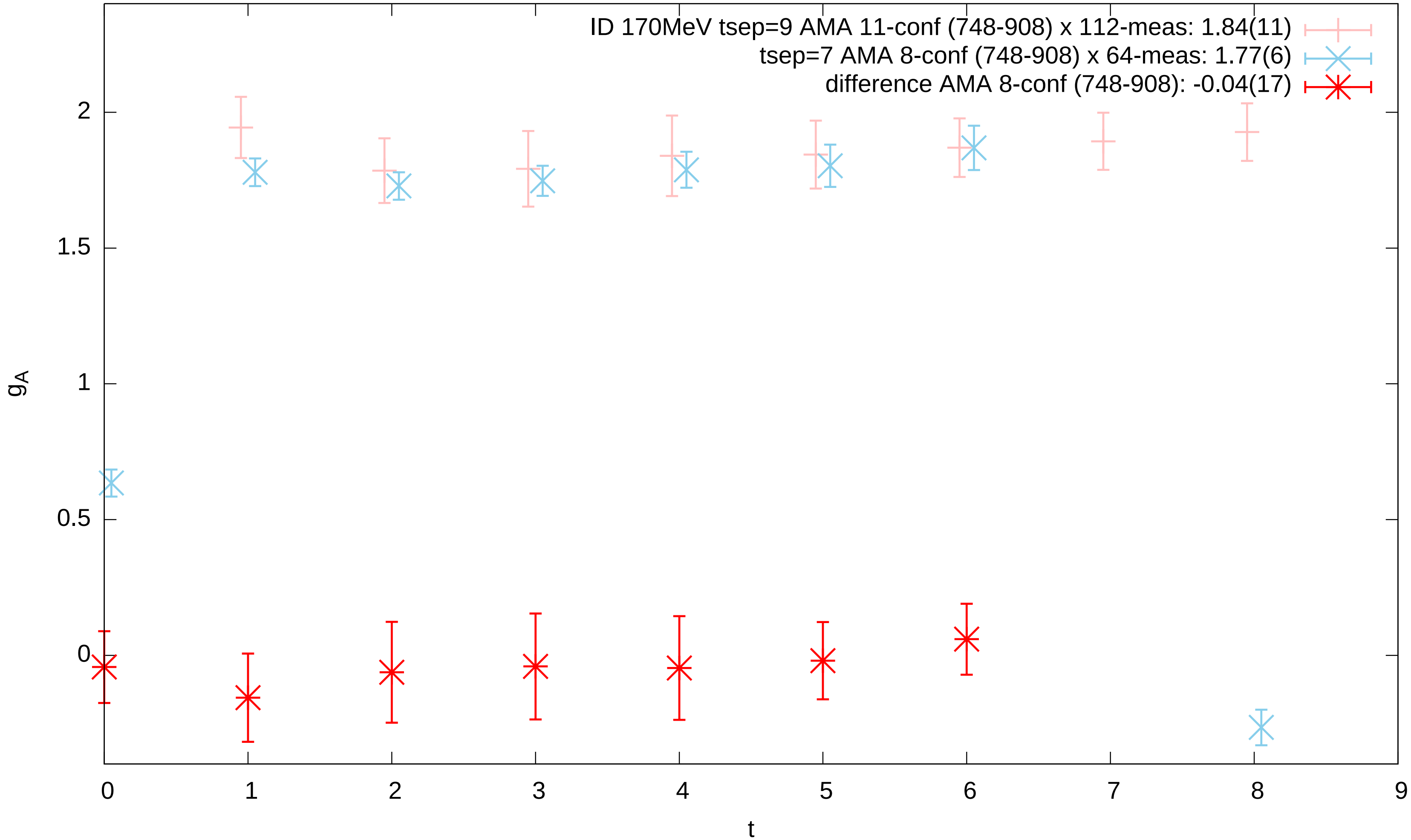

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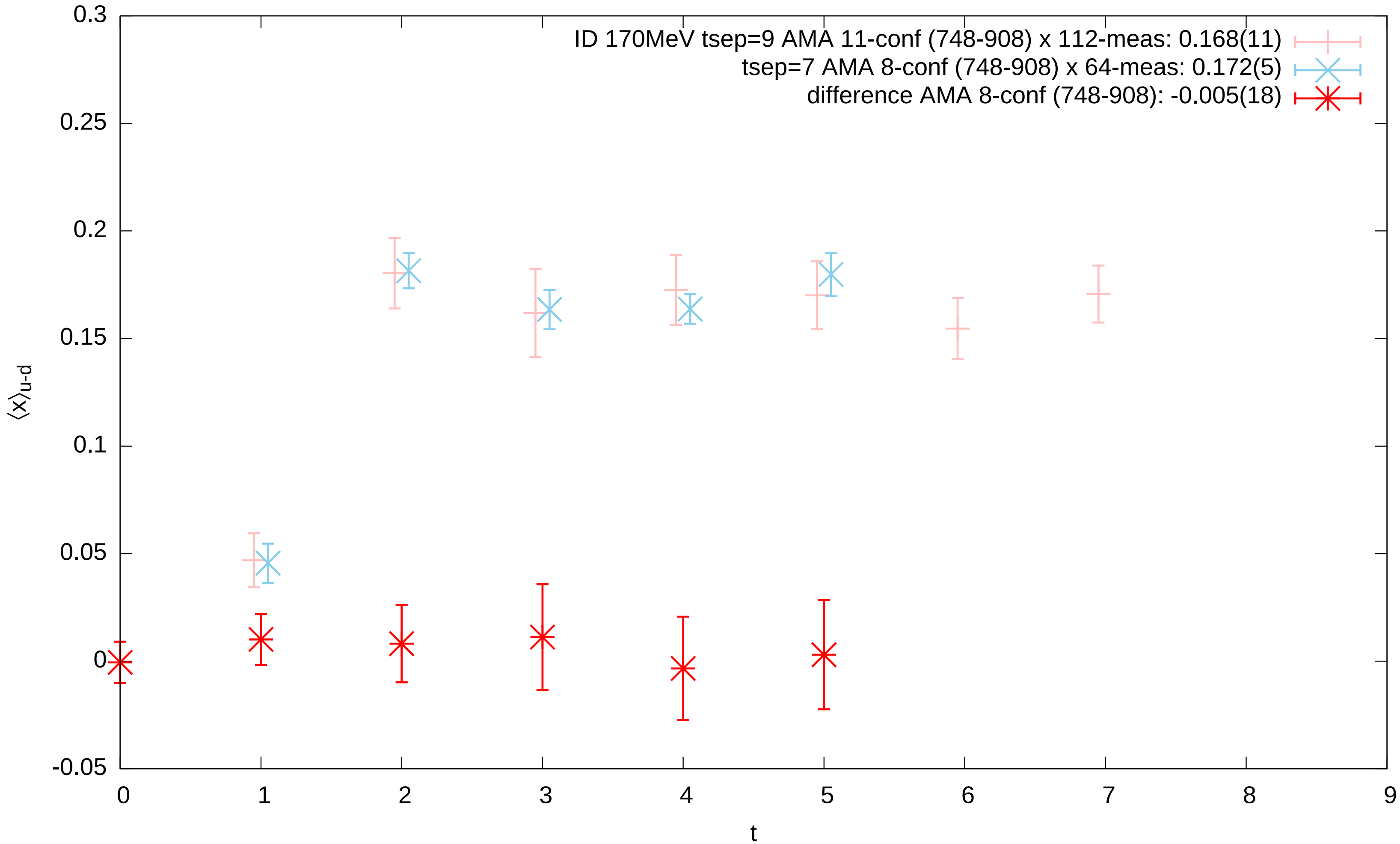

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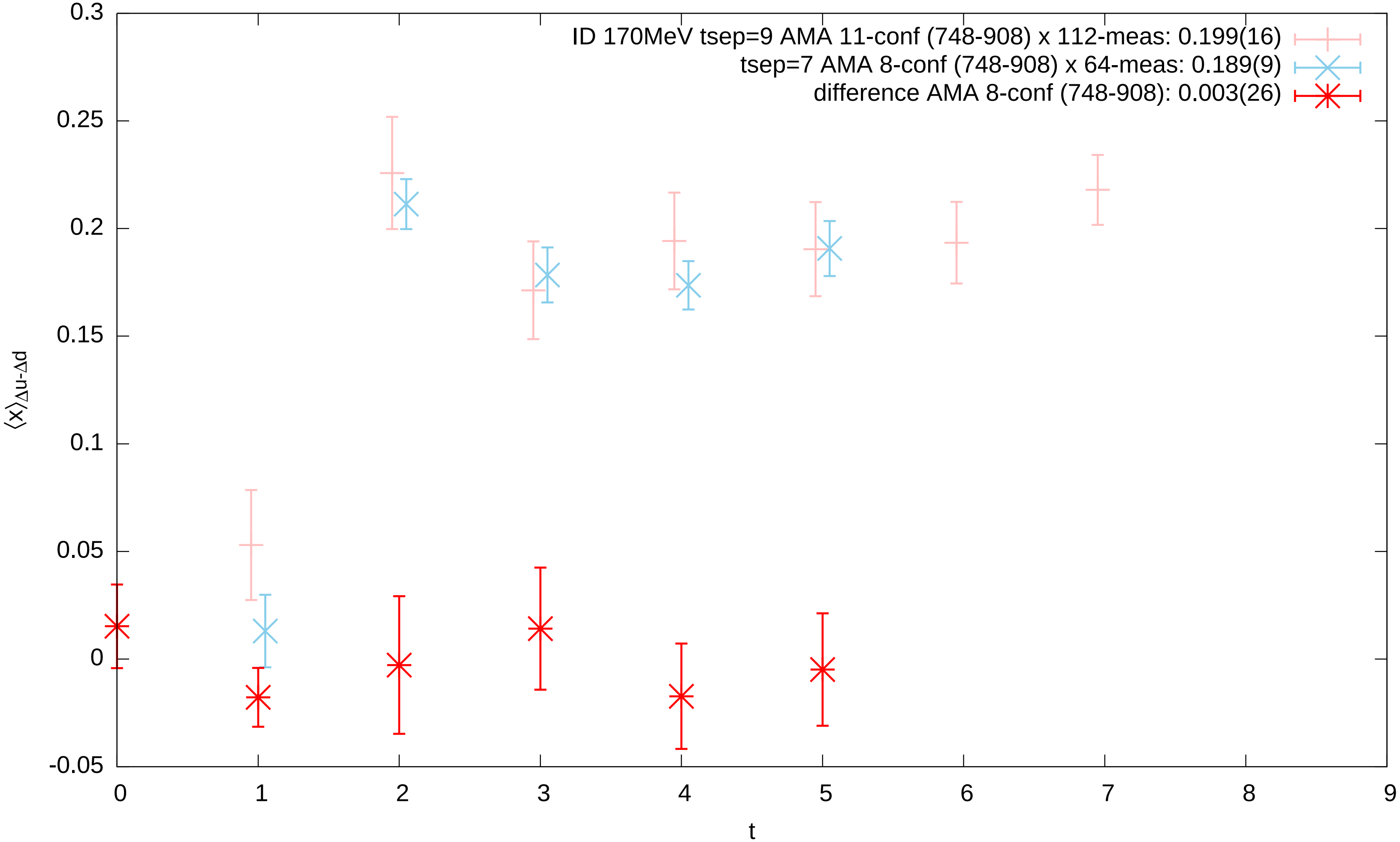

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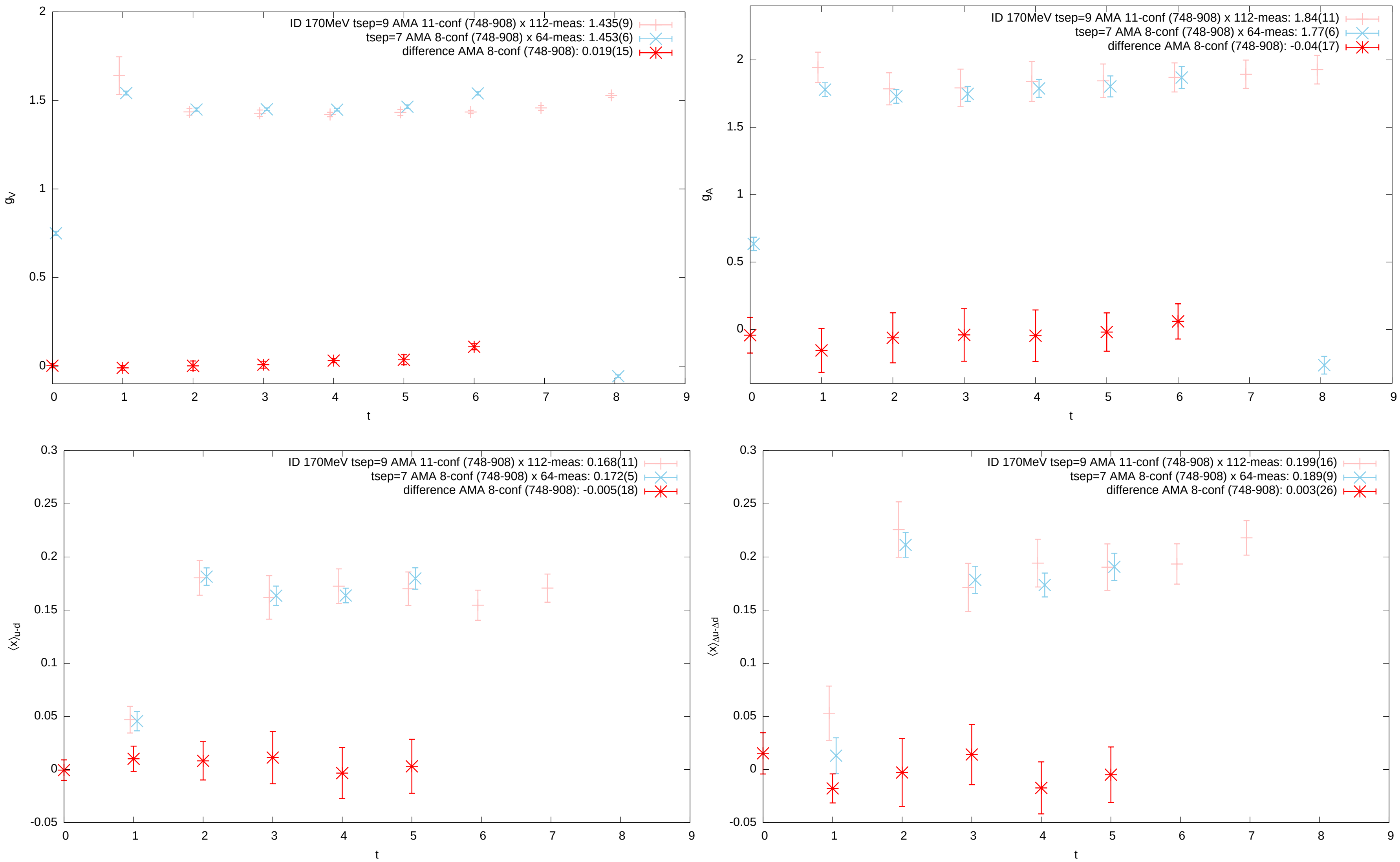

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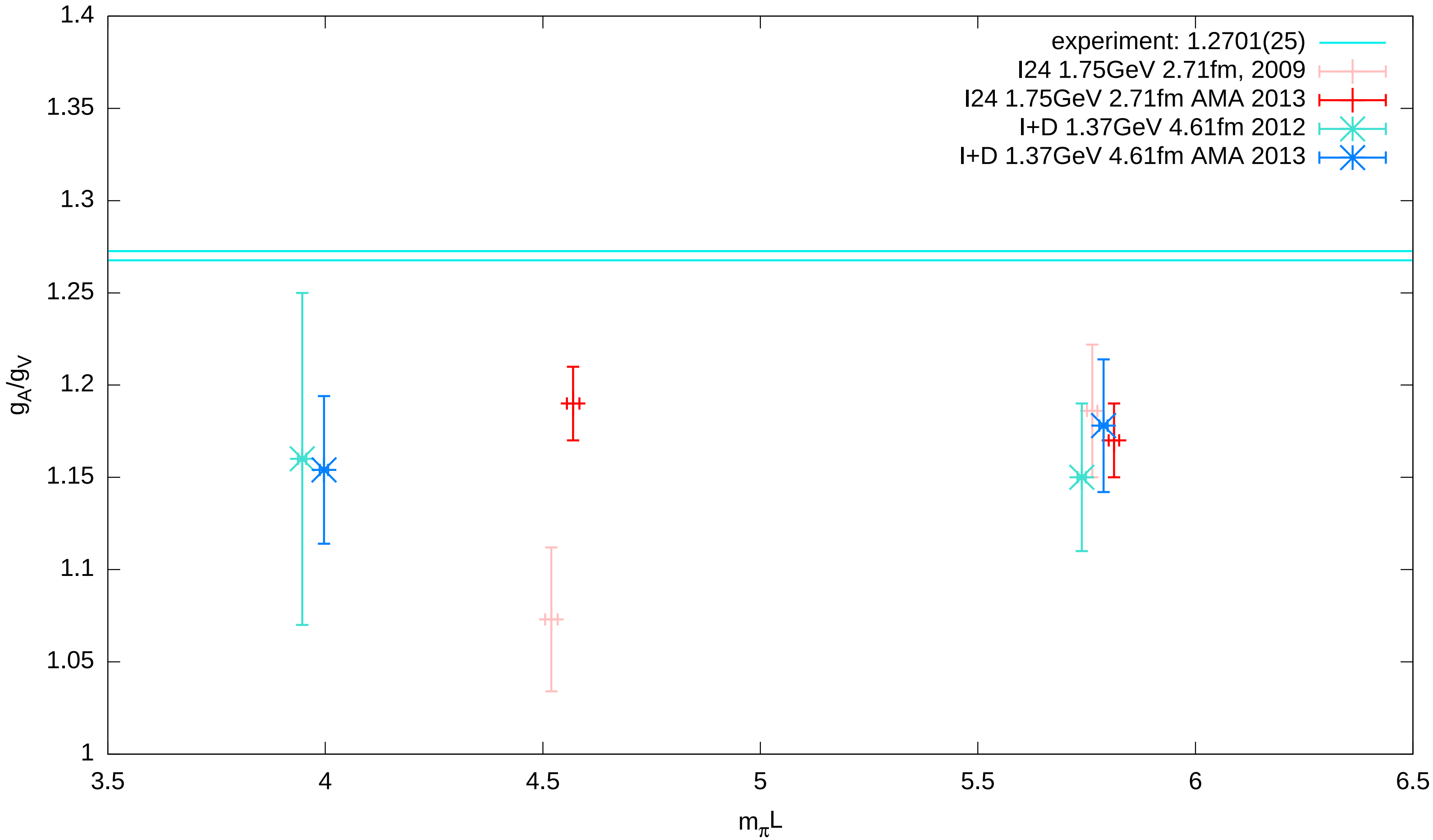
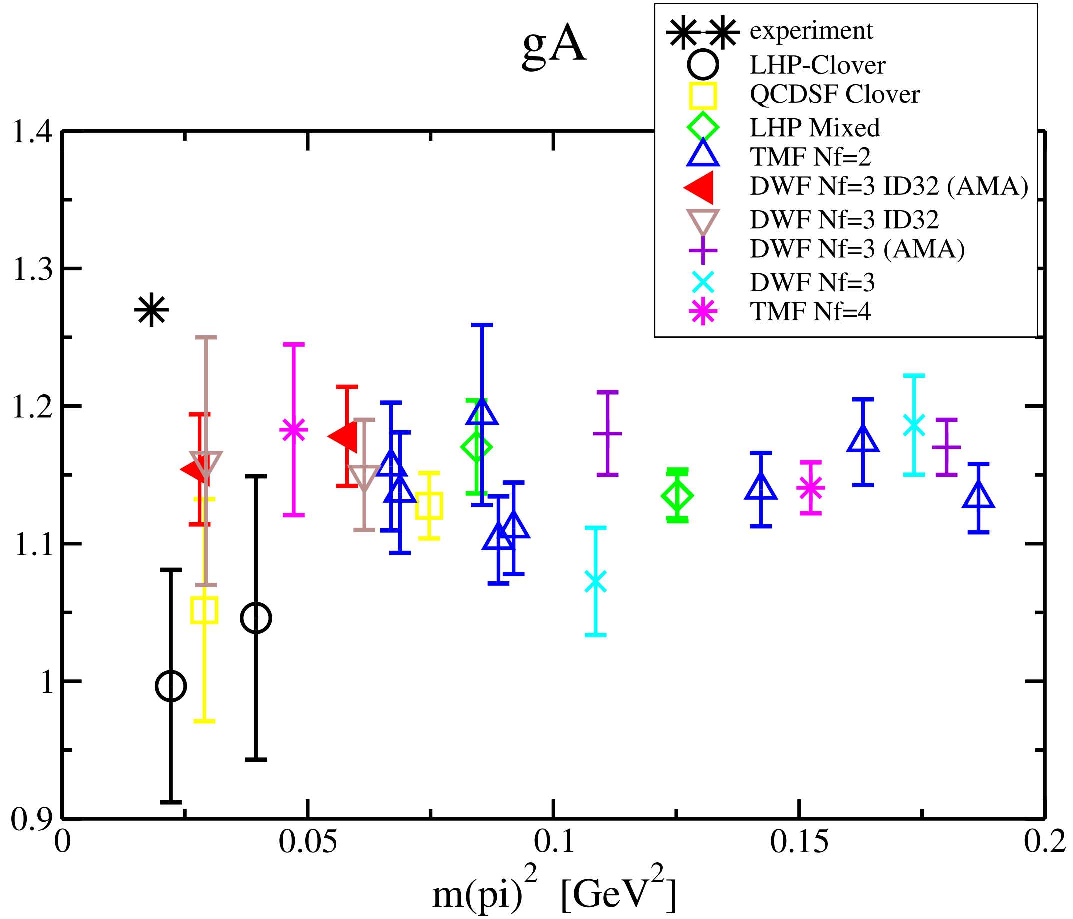


Excited-state contamination now is unlikely the cause.

Almost 5-standard-deviation significance at $m_\pi L \sim 5.8$.

Appears like monotonically decreasing with $m_\pi L$.

About 10-% deficit in g_A/g_V seems in solid agreement with majority of other calculations:



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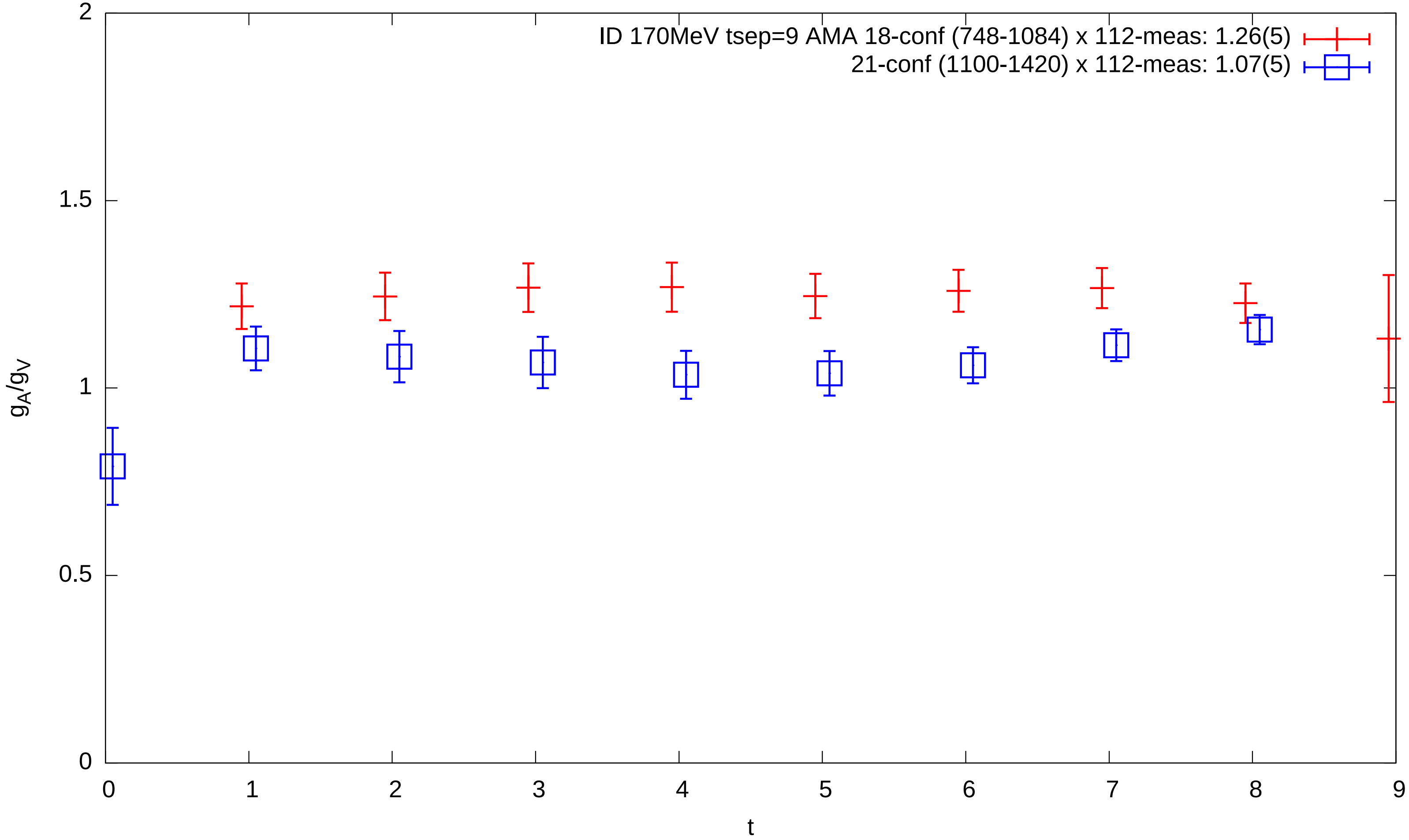
Long-range auto-correlation seen in g_A/g_V at $m_\pi = 170$ MeV:



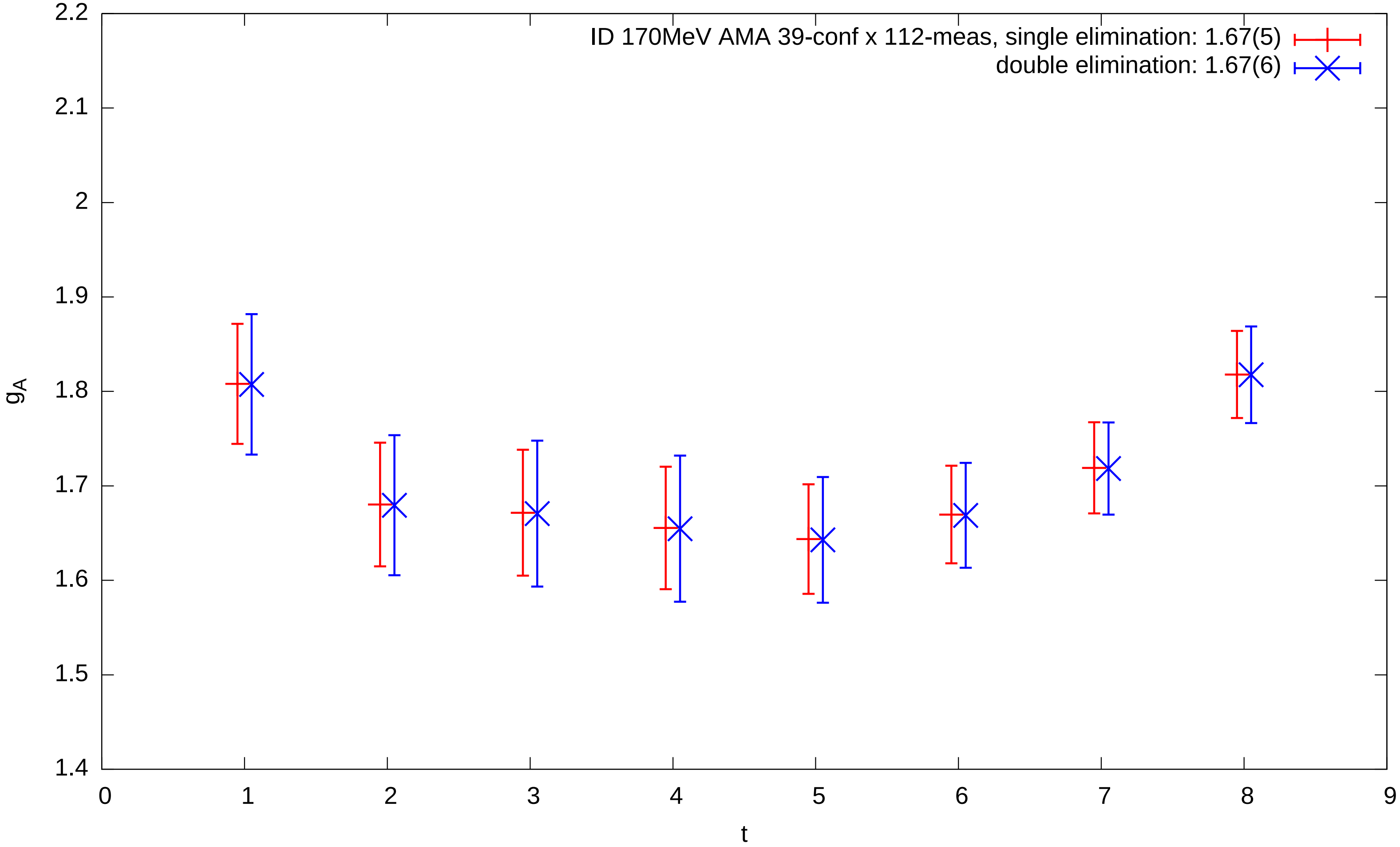
Indicative of insufficient spatial volume.

Systematics other than the spatial volume have been more or less dismissed, in particular the excited states.

$L > 8$ fm is likely required at the physical point, $m_\pi \sim 140$ MeV, to fully contain g_A and so $g_{\pi NN}$.

Can this be reconciled with conventional nuclear theory with point-like and non-relativistic nucleons?

Indeed the estimated errors grow from single- to double-elimination jack knife for g_A and g_A/g_V:



Two successive configurations, separated by 16-trajectory interval, are almost completely correlated.

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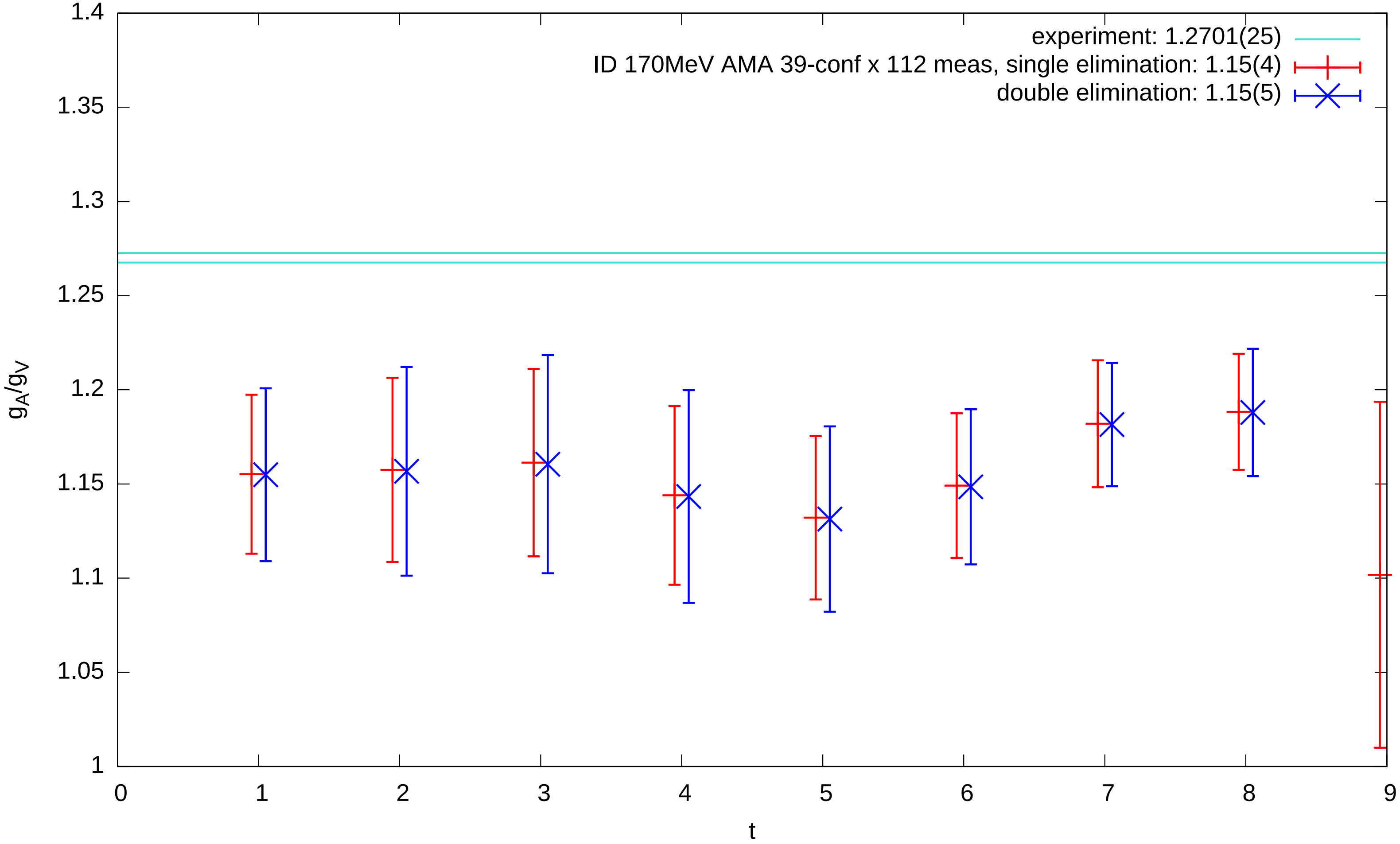
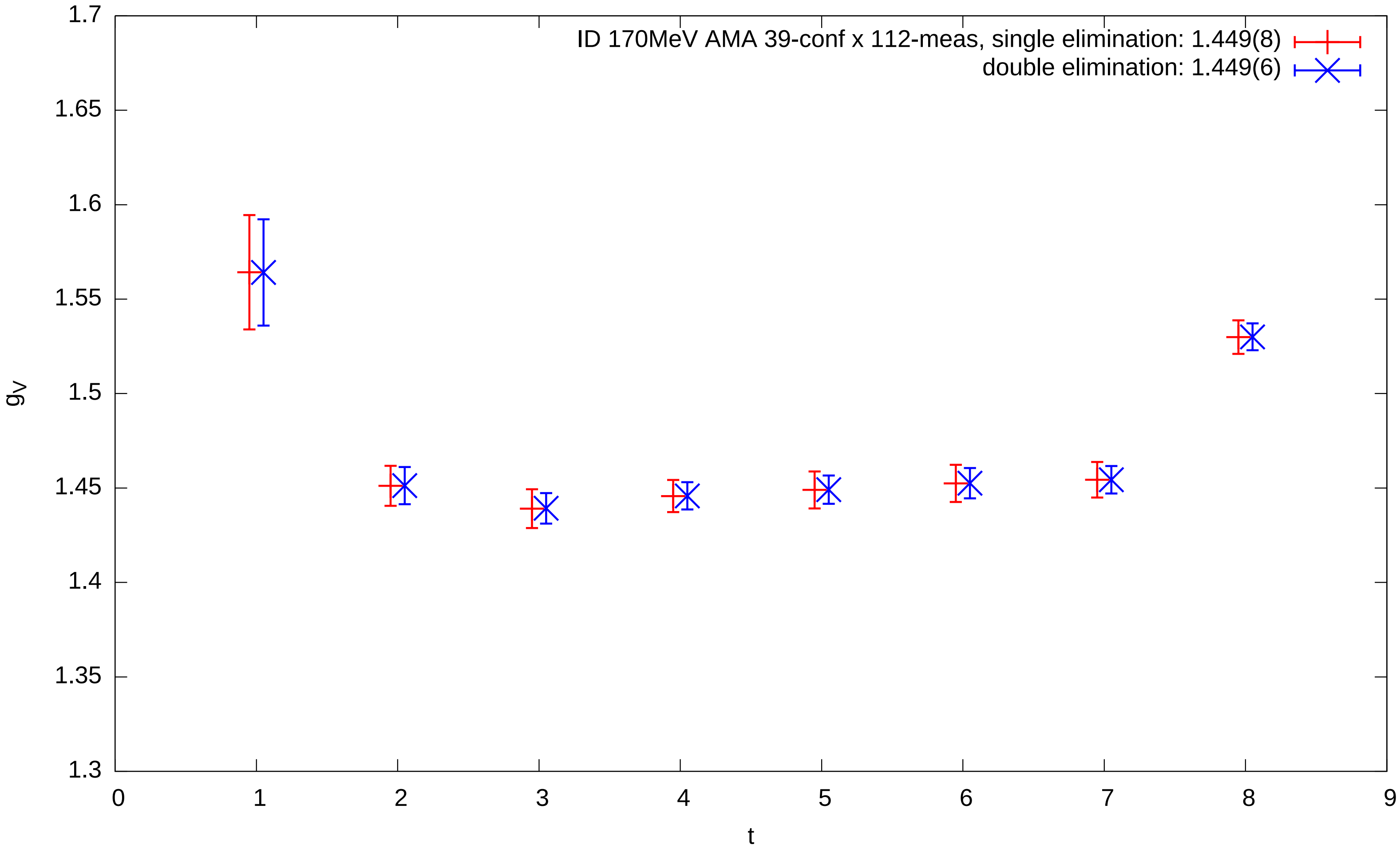


Two successive configurations, separated by 16-trajectory interval, are almost completely correlated.

But no such auto-correlation is seen in other observables, g_V, $\langle x \rangle_{u-d}$ or $\langle x \rangle_{\Delta u - \Delta d}$:



Double-elimination JK sampling does not differ from single-elimination except for g_A.
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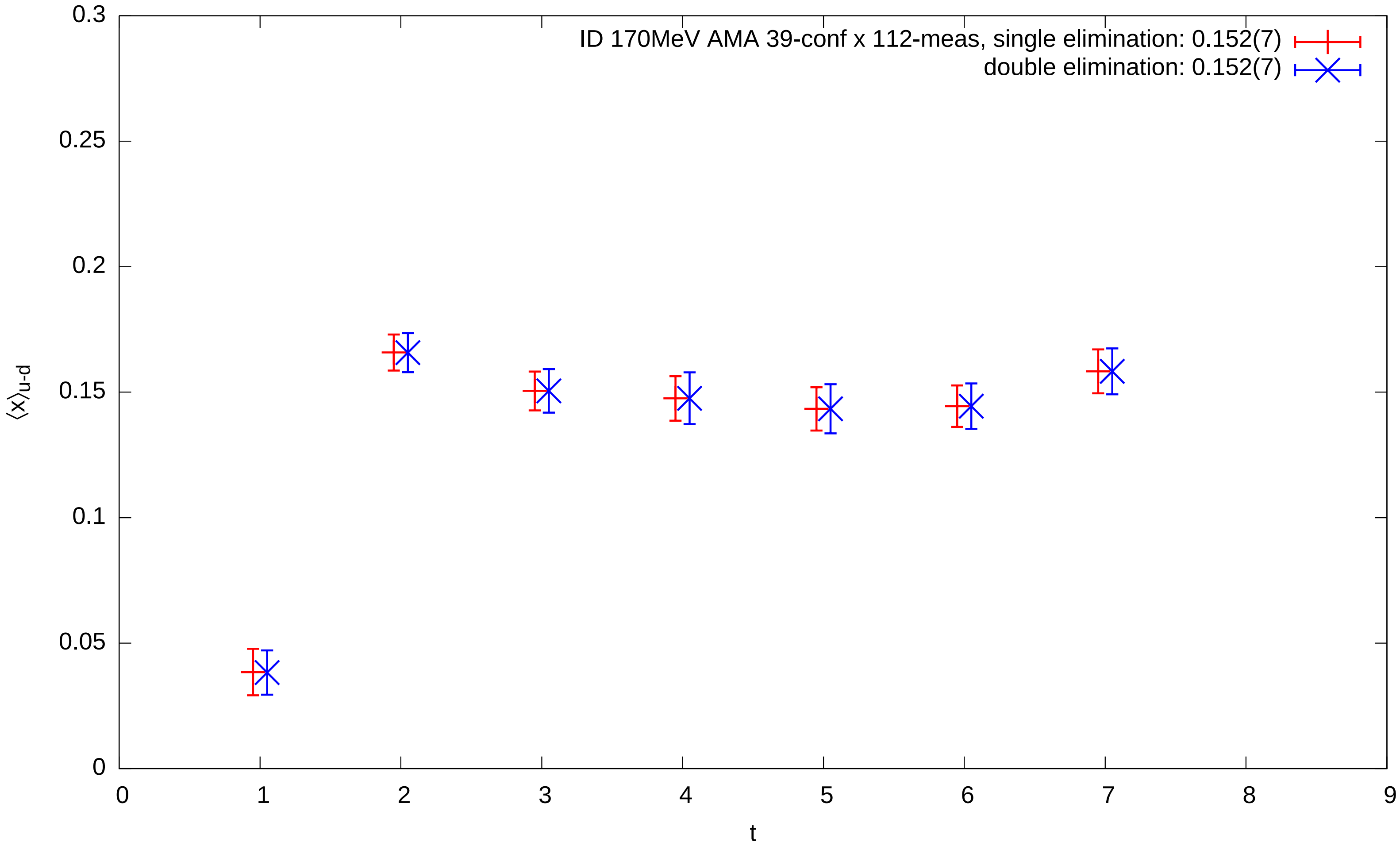

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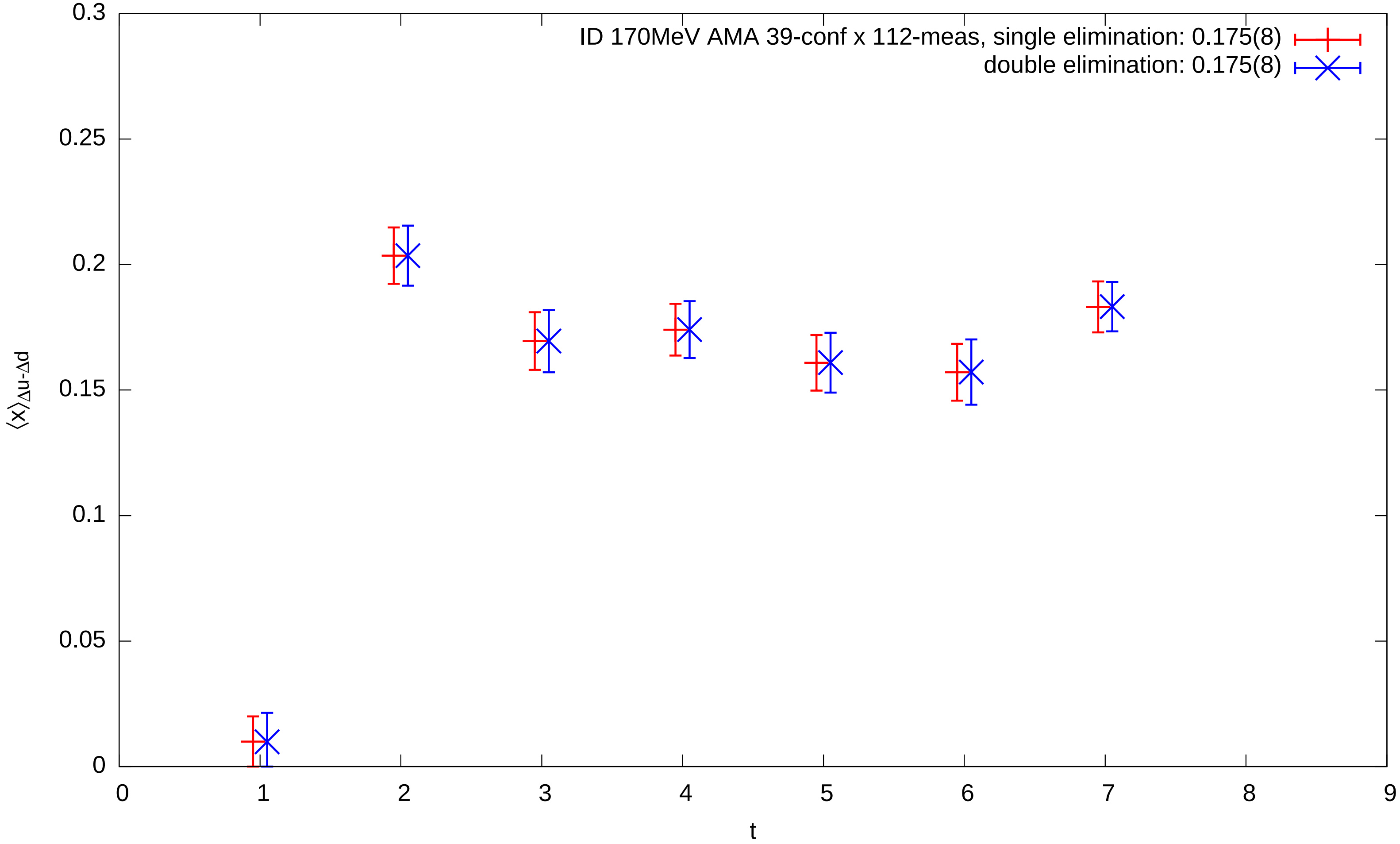

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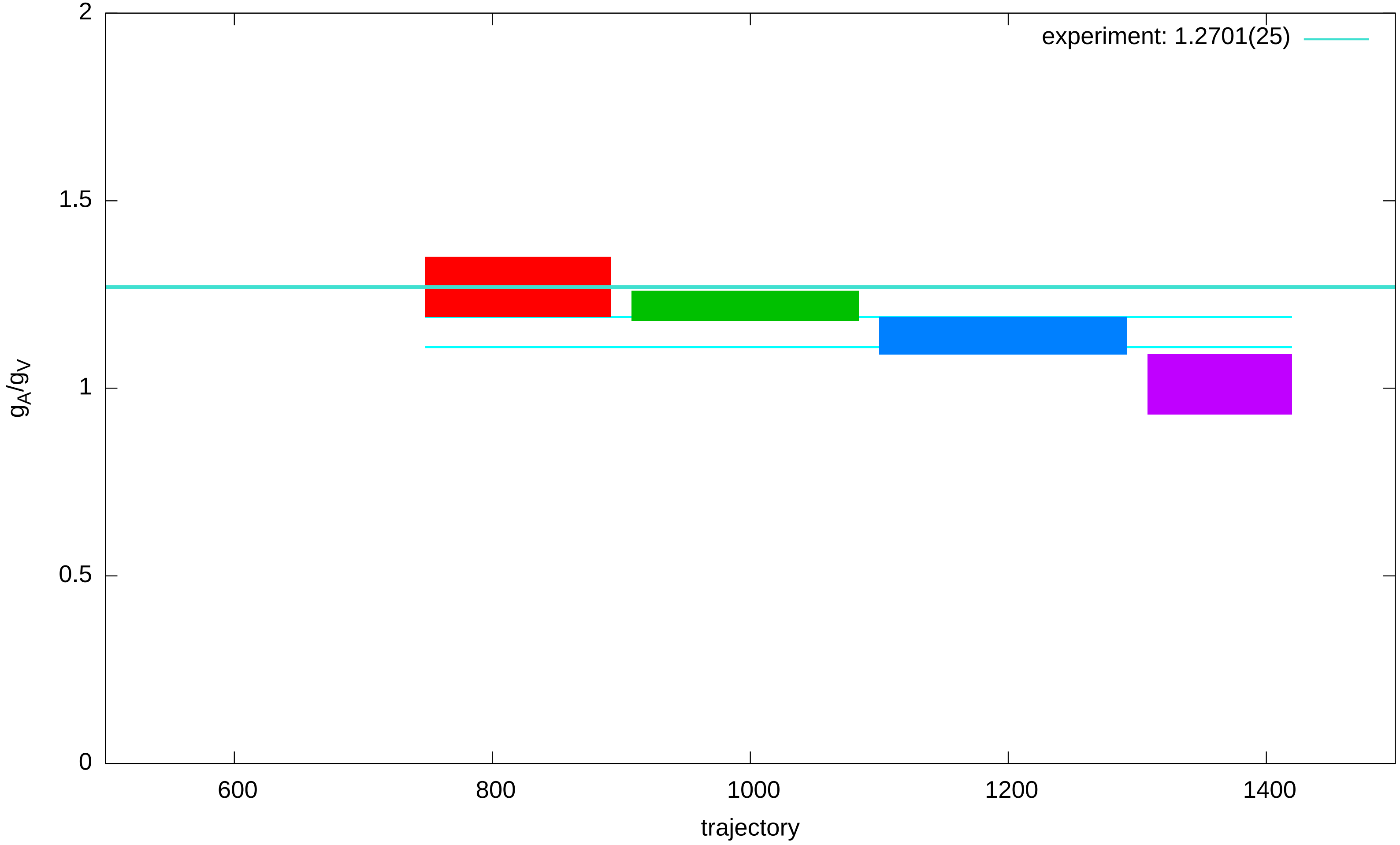
Long-range auto-correlation seen in g_A/g_V at $m_\pi = 170$ MeV:



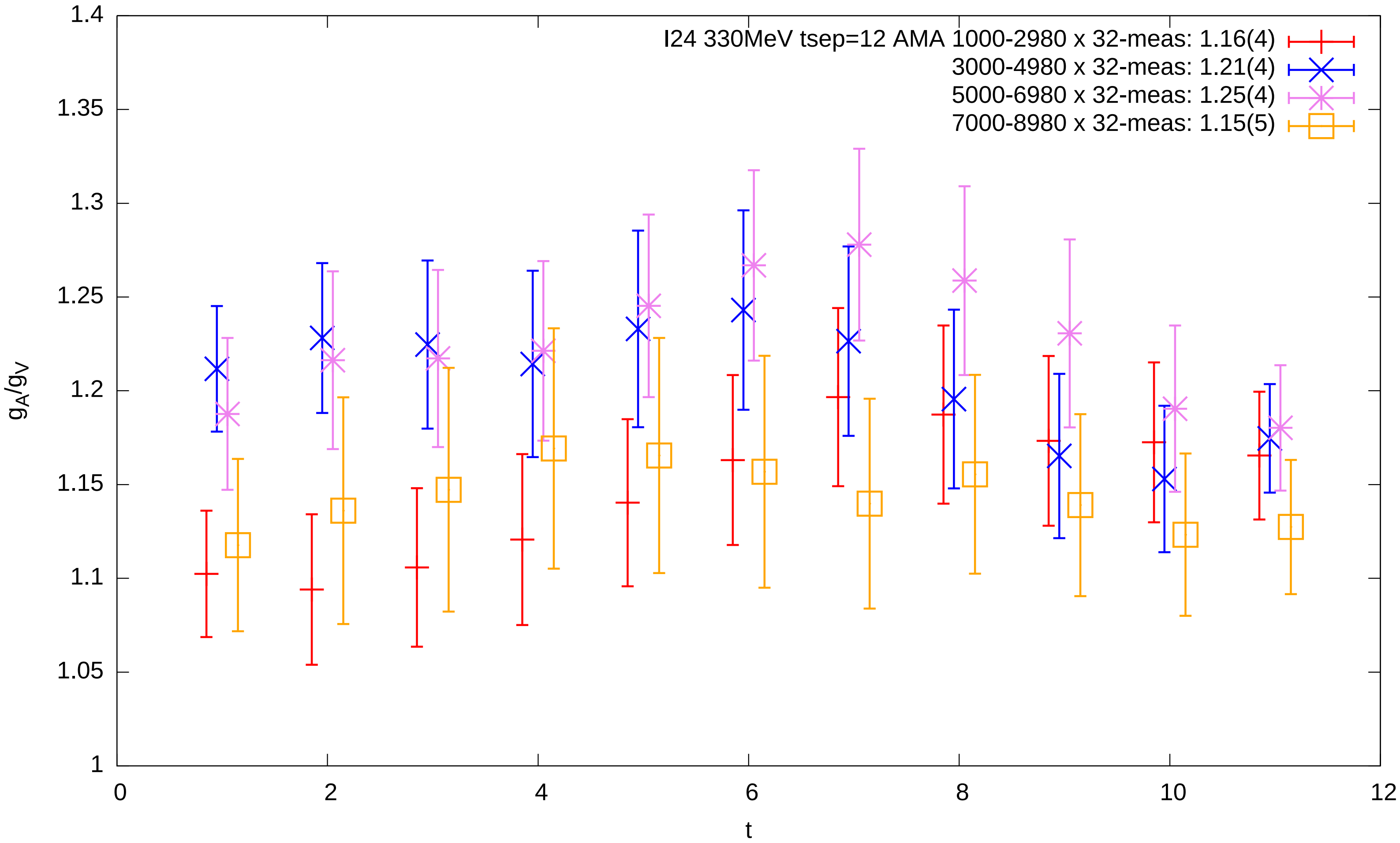
Indicative of insufficient spatial volume.

Systematics other than the spatial volume have been more or less dismissed, in particular the excited states.

$L > 8$ fm is likely required at the physical point, $m_\pi \sim 140$ MeV, to fully contain g_A and so $g_{\pi NN}$.

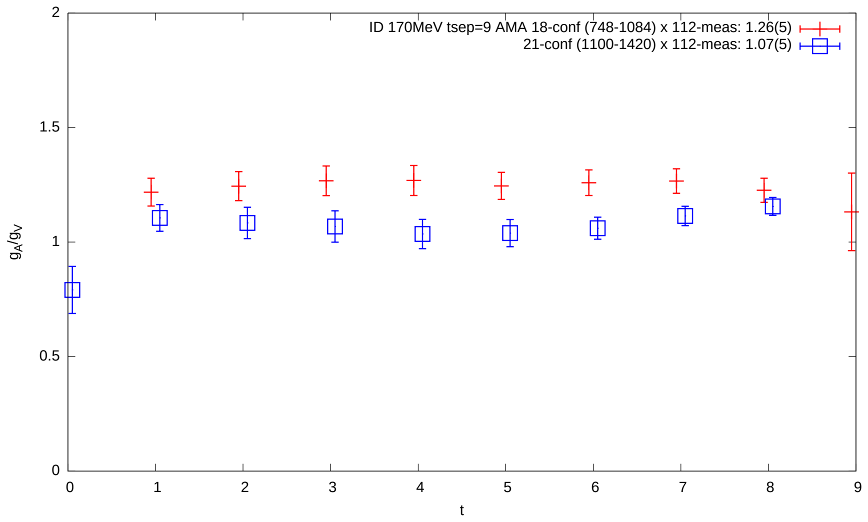
Can this be reconciled with conventional nuclear theory with point-like and non-relativistic nucleons?

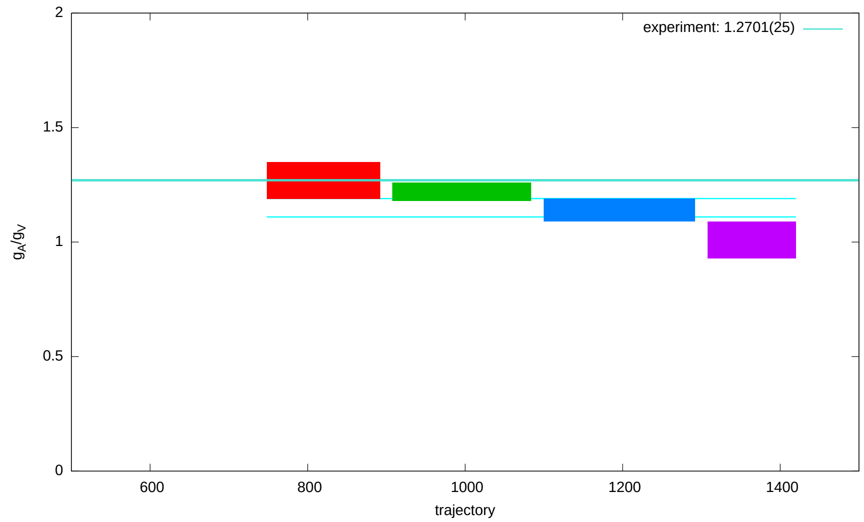
Long-range auto-correlation also seen in g_A/g_V also at $m_\pi = 330$ MeV:



but not at any larger $m_\pi L$.
Indicative of insufficient spatial volume.

Long-range auto-correlation seen in g_A/g_V:





Non-AMA analyses are much noisier but not inconsistent with these:

Indicative of insufficient spatial volume.

Systematics other than the spatial volume have been more or less dismissed, in particular the excited states, except perhaps $O(a^2)$ and isospin breaking.

$L > 8$ fm is likely required at the physical point, $m_\pi \sim 140$ MeV, to fully contain g_A and so $g_{\pi NN}$.

Can this be reconciled with conventional nuclear theory with point-like and non-relativistic nucleons?

Conclusions: RBC+UKQCD work on nucleon structure using the 2+1f dynamical DWF ensembles,

- lattice cutoff $\sim$ 1.4 GeV, $(4.6\text{fm})^3$ spatial volume,
- good chiral and flavor symmetries up to $O(a^2)$, $m_{\text{res}}a \sim 0.002$,
- $m_\pi \sim 170$ and 250 MeV, $m_N \sim 0.98$ and 1.05 GeV.

Successful deployment of the AMA technique resulted in 10-20 times more efficient collection of statistics:

No excited-state contamination is seen in 2+1-flavor dynamical DWF RBC+UKQCD ensembles:

- about 10-% deficit in g_A/g_V seems solid except perhaps for $O(a^2)$ error, 3–5 standard deviation significance,
- long-range autocorrelation is seen in g_A, but not anything else,
- not inconsistent with the RBC-conjecture of finite-size scaling with $m_\pi L$,
- hinting at the first concrete evidence for the pion cloud surrounding nucleon.
- At physical m_π, volumes larger than $(\sim 8\text{fm})^3$ seem necessary:

Nucleon is hardly point-like: How does this reconcile with the conventional nuclear models?

Signals for the isovector form factors and low moments of structure functions are solid.

Now we are starting to calculate at physical mass!